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SMITHSONIAN INSTITUTION
ASTROPHYSICAL OBSERVATORY

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## SPECIAL REPORT

Number 161

# DIFFERENTIAL ORBIT IMPROVEMENT (DØI-3)

by


E. M. Gaposchkin





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"A slow sort of country!" said the Queen. "Now, here, you see, it takes all the running you can do, to keep in the same place. If you want to get somewhere else, you must run at least twice as fast as that!"

THROUGH THE LOOKING GLASS
Lewis Carroll

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# DIFFERENTIAL ORBIT IMPROVEMENT (DØI-3)[1]

by

E. M. Gaposchkin[2]

## Introduction

It is probably more accurate to look upon my task of writing this volume as that of an editor rather than an author. I am not collecting various related works, though I must quote from several and must refer to many other sources throughout. Differential Orbit Improvement, DØI, actually consists of many programs, each one evolving from the previous, and it clearly shows the thumbprints of many people. Dr. George Veis originated the concept of what the program should do. Though his design was for the study of geodetic systems (i.e., determination of station coordinates), the concept is so general that the program also worked splendidly for orbit analysis and thus became the backbone of the operational tracking. Dr. Yoshihide Kozai supplied the orbit theory (short-period terms) and also an a posteriori confirmation of the validity of the choices of particular coordinate systems best suited for orbit analysis. Charles Moore was the first programmer. The first published description of the program (JPL) was a joint effort, carried out by Moore and Veis. Much of Moore's original organization and coding remains intact today. Later versions have additions proposed by Veis, Kozai, Imre Izsak, Dr. Luigi G. Jacchia, and Jack W. Slowey. The early mathematical documentation is still valid today, as we have purposely required that changes and modifications in the operation of the program not alter the basic theoretical definition of the orbital parameters (for instance by redefining the orbit theory). My task in this report is therefore to completely document the program in existence today. No doubt further changes will be made, and possibly this write-up will be out of date by the time it is published!

It seems obvious to us, looking back over the early years of the space age, that a program of this sort was inevitable. The phenomenological determination of time-dependent parameters is a powerful tool, used in much if not all of the early work with satellites. It is fortunate indeed that Veis, Moore, Kozai, and the others were on hand, as each contributed greatly to this program. I must also mention Mrs. Barbara Rush, Mrs. Judith Frommer, and Henry Wadzinsky, who were instrumental in programming and checking out the DØI-2, and Miss Sandra Howard and Miss Karen Johnson, who helped with the current DØI-3.

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[1]This work was supported in part by grant NsG 87-60 from the National Aeronautics and Space Administration.

[2]Formerly, Chief, Computations Division; presently with Research and Analysis, Smithsonian Astrophysical Observatory.

In this report I hope to cover all the aspects of the program. While I hope the entire report will be of interest, I have written each section to stand alone. The reader who is only generally interested in the technique can benefit from reading the general description and philosophy of the program. Those who wish also to interpret the results will find the complete mathematical formulation of interest. The flow charts and description of the "how" of the program will be useful to anyone interested in finding out how a certain expression was applied or contemplating writing his own program. The operating description is probably the most valuable section of this report, as it gives instructions for using the program. We hope that the organization of the program is general enough that future changes will not affect decks already set up.

My aim in writing this report has been twofold: to describe the subject so that the neophyte will find it helpful, and to maintain as well the interest of the more experienced reader.

## Reduction of Observations

A large segment of the program is concerned with reducing observations to a uniform system. (Internally the program uses direction cosines.) Part of the program's strength is its ability indiscriminately to combine many kinds of observations in the same run. This is true in most applications, and I have attempted to document carefully all cases mentioned in this report in which it is not true.

In right ascension ($\alpha$) and declination ($\delta$).--The program uses the following procedure to reduce observations in right ascension and declination:

OBTY = 0

(1) Convert to direction cosines

$$\ell = \cos\delta\cos\alpha; \quad m = \cos\delta\sin\alpha; \quad n = \sin\delta. \tag{1}$$

(2) Rotate to defined sidereal system

$$\bar{\ell}_0 = P_2P_1\bar{\ell}\ ; \qquad \bar{\ell} = \begin{pmatrix}\ell\\ m\\ n\end{pmatrix}. \tag{2}$$

See coordinate systems for $P_2P_1$.

(3) Optional rotation

$$\bar{\ell}_0 = P_3P_2'P_1\bar{\ell}\ . \tag{3}$$

This is discussed on page 48, under "RØTI, RØTII control cards."

The notation "OBTY = " refers to the code punched in column 56 of the observation card. This code and the card format are defined in the section headed "Observation Cards," beginning on page 34.

$P_1$ converts to epoch of 1950.0 from the epoch of the star catalog (see figure 1).

$P_2$ converts to current equator (see figure 2).

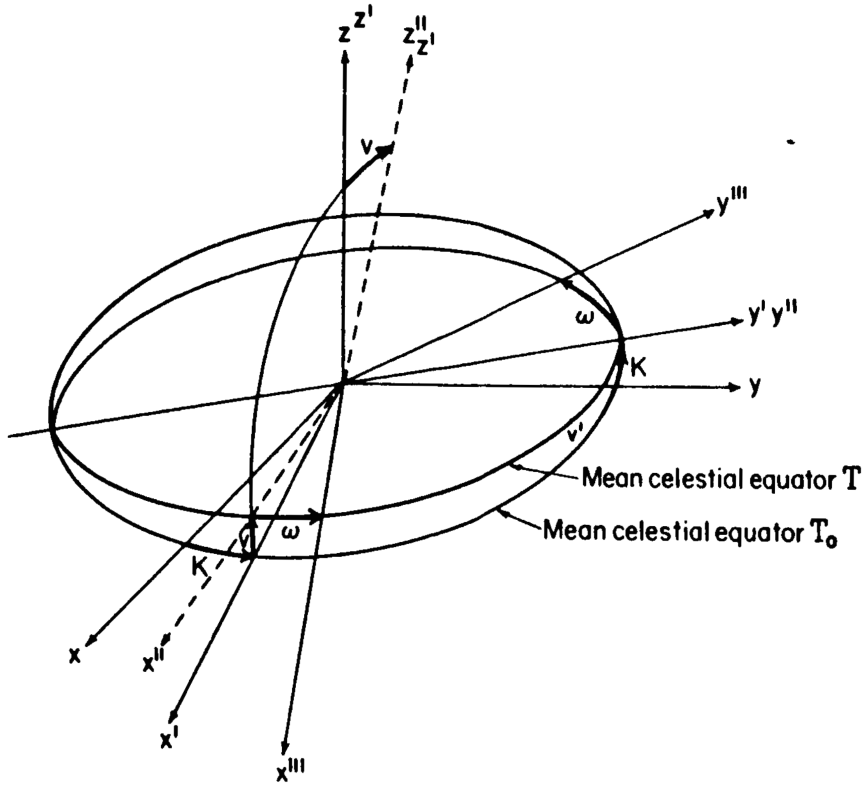


Figure 1.--$P_1$ rotation to the equinox of 1950.0

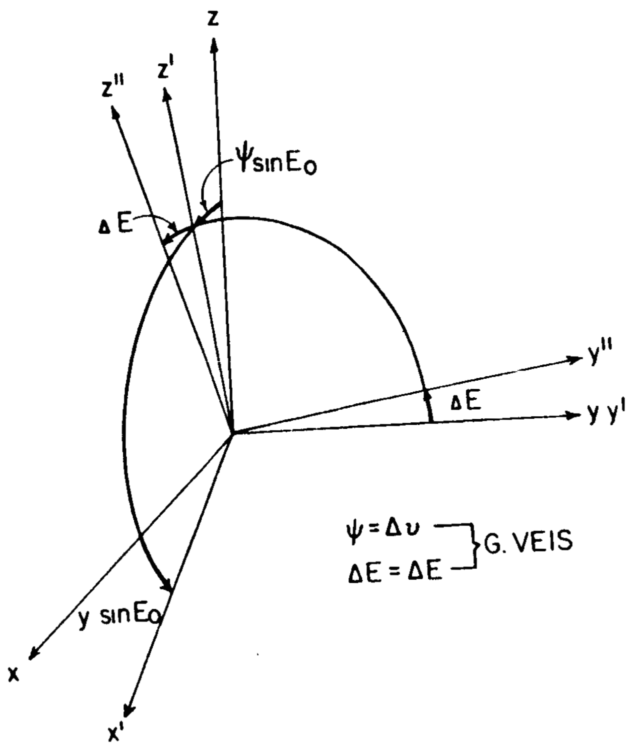


Figure 2.--$P_2$ rotation to the current equator.

Rotation from equinox $T_0$ to equinox T:

$$P_1 = \begin{pmatrix} -\sin\varkappa\sin\omega + \cos\varkappa\cos\omega\cos v & -\cos\varkappa\sin\omega - \sin\varkappa\cos\omega\cos v & -\cos\omega\sin v \\ \sin\varkappa\cos\omega + \cos\varkappa\sin\omega\cos v & \cos\varkappa\cos\omega - \sin\varkappa\sin\omega\cos v & -\sin\omega\sin v \\ \cos\varkappa\sin v & -\sin\varkappa\sin v & \cos v \end{pmatrix} \quad (4)$$

$$\varkappa = [.11171334E\text{-}3 + .67743016E\text{-}9\ (T_0-1900)]\ (T-T_0) + .14655918E\text{-}9\ (T-T_0)^2 .$$

$$\omega = [.11171334E\text{-}3 + .67743016E\text{-}9\ (T_0-1900)]\ (T-T_0) + .53087098E\text{-}9\ (T-T_0)^2 .$$

$$v = [.97189871E\text{-}4 - .41369151E\text{-}9\ (T_0-1900)]\ (T-T_0) - .20687000E\text{-}9\ (T-T_0)^2 .$$

Angles are in radians, and T is in years; sin $\varkappa$ has 56000-year period; sin $\omega$ has 56000-year period; sin v has 65000-year period. $T_0$ is the equinox of the reference catalog.

Rotation from equator of 1950 to equator of date (t):

$$P_2 = \begin{pmatrix} 1 & 0 & -\Psi\sin E_0 \\ 0 & 1 & -\Delta E \\ \Psi\sin E_0 & +\Delta E & 1 \end{pmatrix} \quad (5)$$

$$\Psi = \Psi_1 T + \Psi_3 \sin A_1 + \Psi_4 \sin 2A_1 + \Psi_5 \sin A_2 + \Psi_6 \sin A_3;$$

$$\Delta E = E_3 \cos A_1 + E_4 \cos 2A_1 + E_5 \cos A_2 + E_6 \cos A_3 ;$$

where

$$A_1 = .211408241 - .924219906E\text{-}3\ t$$

$$A_2 = 3.49349291 + .344055813E\text{-}1\ t$$

$$A_3 = 2.24736972 + .459943057\ t;$$

$$\Psi_1 = .668643158E\text{-}6 \qquad E_0 = .409206212$$

$$\Psi_3 = -.835314579E\text{-}4 \qquad E_3 = .446455222E\text{-}4$$

$$\Psi_4 = .101229097E\text{-}5 \qquad E_4 = -.436332313E\text{-}6$$

$$\Psi_5 = -.616101225E\text{-}5 \qquad E_5 = .267035375E\text{-}5$$

$$\Psi_6 = -.977384380E\text{-}6 \qquad E_6 = .436332313E\text{-}6 .$$

Angles are expressed in radians and t in days since 1950.0.

In principle, the $P_2$ matrix is

$$P_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \Delta E & -\sin \Delta E \\ 0 & \sin \Delta E & \cos \Delta E \end{pmatrix} \begin{pmatrix} \cos(\chi \sin E_0) & 0 & -\sin(\chi \sin E_0) \\ 0 & 1 & 0 \\ \sin(\chi \sin E_0) & 0 & \cos(\chi \sin E_0) \end{pmatrix}$$

$$= \begin{pmatrix} \cos(\chi \sin E_0) & 0 & -\sin(\chi \sin E_0) \\ \sin \Delta E \sin(\chi \sin E_0) & \cos \Delta E & -\sin \Delta E \cos(\chi \sin E_0) \\ \cos \Delta E \sin(\chi \sin E_0) & \sin \Delta E & \cos \Delta E \cos(\chi \sin E_0) \end{pmatrix} .$$

In fact, the program uses the following matrix, which is accurate to one part in $10^8$:

$$P_2 = \begin{pmatrix} 1 & 0 & -\chi \sin E_0 \\ \phi\, 10^{-4} \times \phi\, 10^{-4} \cong 0 & 1 & \Delta E \\ \chi \sin E_0 & -\Delta E & 1 \end{pmatrix} .$$

For the optional rotation $P_2'$ and $P_3$ are

$$P_2' = \begin{pmatrix} 1 & -\Psi_0 \cos E_0 & -\Psi \sin E_0 \\ \Psi_0 \cos E_0 & 1 & -\Delta E \\ \Psi \sin E & \Delta E & 1 \end{pmatrix} .$$

$$\Psi_0 = \Psi_0 \sin A_1 + \Psi_4 \sin 2A_1 + \Psi_5 \sin A_2 + \Psi_6 \sin A_3 .$$

$$P_3 = \begin{pmatrix} \cos M & \sin M & 0 \\ -\sin M & \cos M & 0 \\ 0 & 0 & 1 \end{pmatrix} .$$

$$M = \Psi_0 \cos E + \mu_1 T + \mu_2 T^2.$$

$$\mu_1 = 6.1191868E\text{-}7.$$

$$\mu_2 = 1.854875E\text{-}12.$$

$$T = \text{days from } 1950.0.$$

In direction cosines $\ell$, m, n.--Observations in direction cosine are reduced in the following manner:

(1) Take topocentric measurement of $\ell$, m, and

OBTY = 4, 5

$$n = \sqrt{1 - \ell^2 - m^2} \ . \tag{6}$$

(2) If the observation is uncorrected for refraction, compute the altitude and azimuth from

OBTY = 5

$$h = \sin^{-1} n \ ;$$

$$z = \cos^{-1} \frac{m}{\cos h} \ . \tag{7}$$

Actually, the program deals directly with sin h, cos h, sin z, and cos z from

$$\sin h = n \ ;$$

$$\cos h = \sqrt{\ell^2 + m^2} \ ;$$

$$\sin z = \ell / \sqrt{\ell^2 + m^2} \ ;$$

$$\cos z = m / \sqrt{\ell^2 + m^2} \ .$$

(3) Continue as with azimuth-altitude (z,h), defined immediately below.

In azimuth (z) and altitude (h).--By the following procedure the program reduces observations in azimuth and altitude:

OBTY = 1, 3

(1) Correct for refraction:

OBTY = 3

$$h = h + K \cot h . \qquad \begin{array}{l} K = -.0015, \text{ radio observation;} \\ K = -.00029, \text{ optical observation.} \end{array} \tag{8}$$

(See codes for identification.)

(2) Convert to direction cosines:

$$\begin{aligned} \ell &= \cos h \sin z ; \\ m &= \cos h \cos z ; \\ n &= \sin h . \end{aligned} \tag{9}$$

(3) Rotate to defined sidereal system:

$$\vec{\ell}_0 = A\vec{\ell} , \tag{10}$$

where

$$A = \begin{pmatrix} -(\sin\lambda\cos\theta + \cos\lambda\sin\theta) & -(\cos\lambda\cos\theta - \sin\lambda\sin\theta)\sin\Phi & (\cos\lambda\cos\theta - \sin\lambda\sin\theta)\cos\Phi \\ (\cos\lambda\cos\theta - \sin\lambda\sin\theta) & -(\sin\lambda\cos\theta + \cos\lambda\sin\theta)\sin\Phi & (\sin\lambda\cos\theta + \cos\lambda\sin\theta)\cos\Phi \\ 0 & \cos\Phi & \sin\Phi \end{pmatrix} .$$

Note that

$$(\sin\lambda\cos\theta + \cos\lambda\sin\theta) = \sin(\lambda + \theta);$$

$$(\cos\lambda\cos\theta - \sin\lambda\sin\theta) = \cos(\lambda + \theta);$$

therefore,

$$A = \begin{pmatrix} -\sin(\lambda+\theta) & -\cos(\lambda+\theta)\sin\Phi & \cos(\lambda+\theta)\cos\Phi \\ \cos(\lambda+\theta) & -\sin(\lambda+\theta)\sin\Phi & \sin(\lambda+\theta)\cos\Phi \\ 0 & \cos\Phi & \sin\Phi \end{pmatrix} . \tag{11}$$

$\Phi$ = astrometric latitude of station (altitude of north celestial pole);
$\lambda$ = longitude of station;
$\theta$ = sidereal time.

In range (r).--To reduce observations in range the procedure is

(1) When r is observed in conjunction with direction, use

OBTY = 0,1,3,4,5

$$\vec{x}_0 = r\ell_0 . \tag{12}$$

(2) When r is observed alone use

OBTY = 6

$$r_0 = r . \tag{13}$$

## Time Systems*

The whole subject of the relationships between various coordinate systems and time systems is too complicated and lengthy to be treated here in detail. Some idea of the basic principles is, however, very important for the interpretation and understanding of the results. The interested reader is referred to Veis (1963), Newcomb (1960), and Chauvenet (1960) for further considerations on this matter.

The independent variable in orbit theories is called ephemeris time--a uniform time system that cannot be realized in practice. The classical method of measuring time is based on the Earth's rotation as a clock. The rate of rotation varies, however, as a result of effects of the Earth's oblateness, precession, and the attraction of the moon, among other things. Time as defined by the rotation of the Earth is called Universal Time (UT). Throughout this discussion we are concerned only with time as defined at the Greenwich meridian. Since time measured in this way is actually a rotation, it is simpler to think of it as an angle between a reference plane and the meridian. This angle is called the sidereal angle.

This instantaneous sidereal angle can be immediately converted to UT. Universal Time corrected for the effects of the motion of the pole is called UT1. UT1 corrected for seasonal variations in the rotation of the Earth is called UT2, which is not really a uniform time system, since the seasonal variation is only an estimated quantity.

The time system used at SAO is called Modified Julian Days (MJD), defined by the formula

$$MJD = JD - 2400000.5 .$$

The mean sidereal time $\bar{\theta}$ is derived in this manner:

$$\bar{\theta} = 100^{\circ}.075542 + 360^{\circ}.985647348\,(T) + 0^{\circ}.2900 \times 10^{-12}\,(T)^2 , \qquad (14)$$

where T is the number of days since 0 Jan 1950:

$$T = (MJD - 33282.0) .$$

Angles are in degrees.

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*I am indebted to Veis (1963) for this section and its equations.

The true sidereal time $\theta'$ is

$$\begin{aligned}\theta' = \bar{\theta} &- 4^{\circ}.392 \times 10^{-3} \sin (12^{\circ}.1128 - 0^{\circ}.052954\ T) && \text{19.1-year period} \\ &+ 0^{\circ}.053 \times 10^{-3} \sin 2(12^{\circ}.1128 - 0^{\circ}.052954\ T) \\ &- 0^{\circ}.325 \times 10^{-3} \sin 2(280^{\circ}.0812 - 0^{\circ}.985647\ T) && \text{solar period} \\ &- 0^{\circ}.050 \times 10^{-3} \sin 2(64^{\circ}.3824 + 13^{\circ}.176318\ T)\ . && \text{lunar period}\end{aligned} \tag{15}$$

This expression is the definition of UT1.

Because atomic time (A1) is presumably closer to ephemeris time than to these time systems, all the precisely reduced observations used at SAO are given in A1 time. Since the orbits are related to UT1, the DØI converts atomic time to UT1. Because there is no analytical expression for this conversion, approximate values are used in the program. These may be modified (see the section on input of constants, beginning on page 31 ).

Since the orbits are computed for only a few weeks at a time, long-period variations in the time systems are unimportant, and changes can be absorbed in the constants of the orbit equations. Care must be exercised only when combining observations in different time systems or when interpreting the orbital elements for physical significance. Precisely reduced observations are identified by the first digit of their observation number, 7. The expression for the modified sidereal angle used in the DØI is

$$\begin{aligned}\hat{\theta} &= 100^{\circ}.075542 + 360^{\circ}.985612288(T) \\ &= 0^{\text{rev}}.277987616 + 1^{\text{rev}}.00273781191(T)\ .\end{aligned} \tag{16}$$

Sidereal time defined in this way is based on a uniformly rotating Earth, with all perturbations averaged out.

33282 = 1 Jan 1950 in MJD.

.277987616 = sidereal angle at 1 Jan 1950 minus nutation in right ascension (revolutions).

1.00273781191 = revolutions per mean solar day for 1900 (Sterne, 1960).

This time system, which has been shown to be the best for orbital analyses (Kozai, 1960), has been fully described by Veis (1963).

This sidereal angle is, then, the angle between the intersection of the equator of date, the equinox of 1950 and the ecliptic, and the mean Greenwich meridian at the time T. In other words, it is the angle between the inertial system in which the satellite's orbit is defined and the geodetic system in which the observing stations (and hence the observations) are defined.

## Orbital Elements

Although in attempting to describe the motion of a satellite in an orbit about the Earth we are dealing with a complex problem (the motion of an object under gravitational attraction from a nonsymmetric body, this motion being affected by atmospheric drag, gravitational attraction by moon and sun, and the sun's radiation pressure), we do use the solution of the idealized two-body problem, as given by classical celestial mechanics, as a first approximation. We then take the other effects into account by introducing perturbations (or small variations) into this first-approximate solution.

We will consider two-body motion in an inertial system with a rotating Earth, in which the simple two-body motion can be represented by six parameters, called elements. We choose three of these elements to be geometric and the remaining three to be dynamic. (This choice is arbitrary; an explanation of the reasons for this particular choice is beyond the scope of this work.)

The satellite's motion occurs in a plane called the orbital plane (see figure 3).

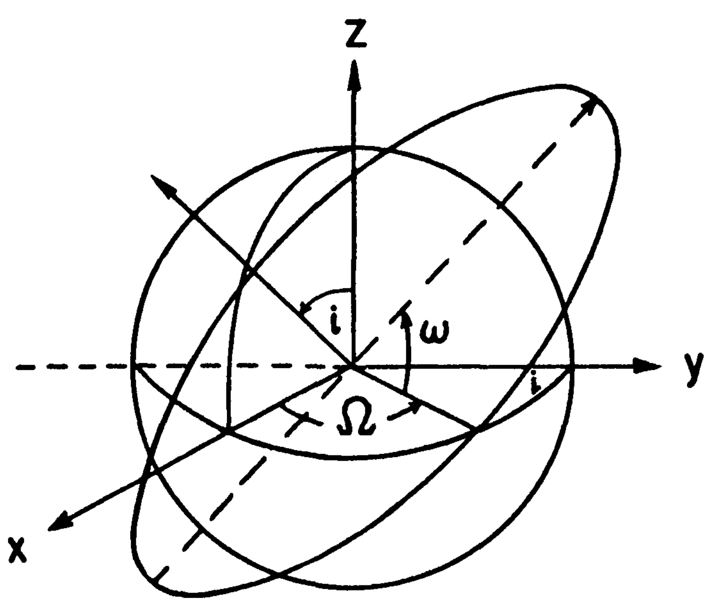



Figure 3.--The orbital plane of a satellite's motion.

The orientation of the orbital plane with respect to the (x,y) plane of the inertial system is described by the two angles $\Omega$ and i. The node, $\Omega$, is the angle between the line of intersection of these two planes and the x-axis; i, the inclination, is defined either as the angle between the normal to the orbit plane and the y-axis or as the angle between the orbit plane and the (x,y) plane (the two are equivalent).

The path of the satellite's motion in the orbital plane is an ellipse, with the center of the Earth as one focus. Perigee is that point on the ellipse closest to this focus. To orient the ellipse in space we must specify the third angle $\omega$, argument of perigee, which is the angle between the line of intersection of the (x,y) plane with the orbital plane and the line joining the perigee and the center of the Earth; $\Omega$, $\omega$, and i are the three geometric elements.

Let us now consider the satellite's motion in the orbital plane (see figure 4).

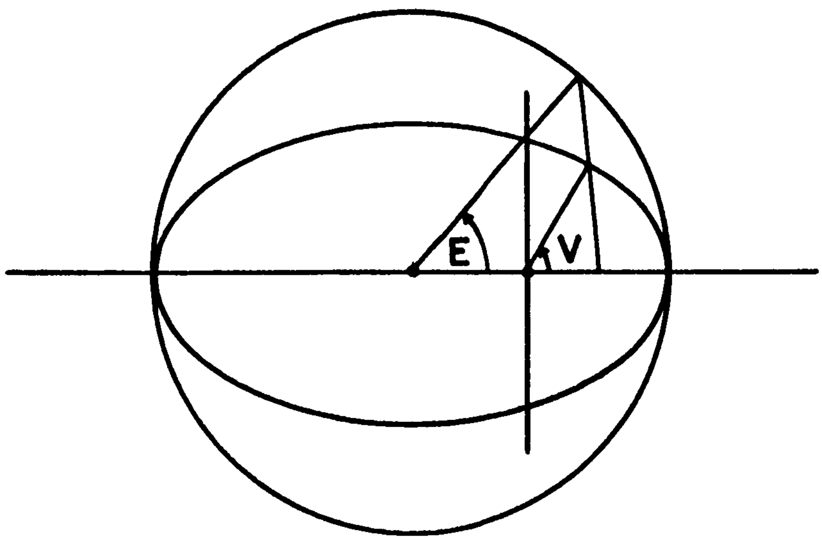



Figure 4.--The path of a satellite's motion in the orbital plane.

The eccentricity, e, of this ellipse will be the fourth element.

The fifth element, M, the mean anomaly, varies linearly with time. To relate it to angular quantities we must solve Kepler's equation, which describes the motion of the satellite in its elliptical orbit in terms of eccentric anomaly, E. We relate E to true anomaly, v, by using the geometric properties of the ellipse.

We state Kepler's equation in the form

$$M = E - e \sin E = n(t - T) , \tag{17}$$

where n is the mean motion:

$$n = \frac{dM}{dt} = \dot{M} \text{ (polynomial part only) .}$$

Equation (17) is the solution of a differential equation with t as the independent variable, so we may choose T as a constant of integration. Thus T, which is called the epoch, is the sixth element.

In the simple two-body problem $\omega$, $\Omega$, i, T, and e are constants, while $M = \text{const.} + n \times t$.

Actually, however, because of the effects mentioned earlier, the orbital motion of an artificial satellite differs somewhat from the idealized motion stated above. Nevertheless, the two motions, the ideal and the real, are related closely enough so that we can express the latter as perturbations in the solution for the idealized case. In fact, we find that by making the six elements time-varying, we can approximate the real orbital motion relatively accurately by

$$\Omega = \Omega(t) ;$$
$$\omega = \omega(t) ;$$
$$i = i(t) ;$$
$$e = e(t) ;$$
$$M = M(t) .$$

We will specify these functions of time as polynomials in t, with optional trigonometric and hyperbolic terms:

$$f(t) = \sum_{i=0}^{n} P_i t^i + \sum S_0 \sin(S_1 + S_2 t) + \sum H_0 e^{H_2(\ell n(H_1 - T))} . \qquad (18)$$

Since we now have the elements as functions of time, we shall classify them at a particular time as instantaneous elements.

The five elements listed above, the mean elements, together with the epoch describe the complete motion of the satellite.

Another point to consider when describing a satellite's motion is the physical size of the orbit. This is usually defined by calculating the semimajor axis a of the orbital ellipse, using the formula

$$a = (k/n^2)^{1/3} ,$$

where $k = g \times m$; $g$ is the constant of gravitation; and $m$ is the mass of the Earth. Because of the Earth's asymmetry, we must add a perturbation in $a$ (see equation 22).

## Computation of Satellite Position

The position of the satellite is computed by means of the following set of equations:

(1) Evaluate the elements at time c:

$$\left.\begin{aligned} \omega &= \omega(t) \\ \Omega &= \Omega(t) \\ i &= i(t) \\ e &= e(t) \\ M &= M(t) \end{aligned}\right\} \quad \text{mean elements} \tag{19}$$

$$n = \dot{M}(t) \quad \text{(polynomial part only)} .$$

(2) Add lunar perturbations optionally:

$$\left.\begin{aligned} \omega &= \omega(t) + \delta\omega \\ \Omega &= \Omega(t) + \delta\Omega \\ i &= i(t) + \delta i \\ e &= e(t) + \delta e \\ M &= M(t) + \delta M \end{aligned}\right\} \quad . \tag{20}$$

Since $\delta a = 0$, there is no contribution to $n$.

These expressions were developed by Izsak (private communication). See appendix 1 for expressions of these lunar perturbations.

Let

$$p = \left(\frac{k}{n^2}\right)^{1/3} (1 - e^2) \tag{21}$$

(semilatus rectum of the mean ellipse).

Then (Kozai, 1959)

$$a = \left(\frac{k}{n^2}\right)^{1/3} \left[ 1 + \frac{1}{3} \frac{J_2}{p^2} \sqrt{1 - e^2} \left(-1 + \frac{3}{2} \sin^2 i\right)\right] ; \tag{22}$$

$$k = .7537172 \times 10^5 \text{ rev}^2 \text{ megameters}^3 \text{ day}^{-2};$$

$$J_2 = .0660546 \text{ megameters}.$$

(3) By iteration solve Kepler's equation for E,

$$E = M + e \sin E \ . \tag{23}$$

(4) Solve for v

$$\sin v = \frac{\sqrt{1 - e^2}}{1-e \cos E} \sin E$$

and (24)

$$\cos v = \frac{\cos E - e}{1-e \cos E} \ .$$

(5) Compute $\cos \ell$ and $\sin \ell$:

$$\ell = \omega + v \ . \tag{25}$$

We apply the second harmonic short-period perturbations as follows:

$$\left.\begin{aligned} \Omega &= \Omega + \delta\Omega \\ \sin i &= \sin i + \delta i \cos i \\ \cos i &= \cos i - \delta i \sin i \\ \sin \ell &= \sin \ell + \delta\ell \cos \ell \\ \cos \ell &= \cos \ell - \delta\ell \sin \ell \\ r_c &= a(1-e \cos E) + \delta R \end{aligned}\right\} \ . \tag{26}$$

The expressions for these perturbations are included in appendix 2.

These perturbations are added to each computed position; since their values are insensitive to small changes in the orbital elements, they are seldom recomputed. The standard error of the least-squares solution is used to determine whether they should be recomputed. When

$$\frac{\sigma_i - \sigma_{i-1}}{\sigma_i} > 0.1 \ ,$$

they are recomputed.

It is difficult to foresee which iterations will cause these short-period terms to be recomputed, but they are always computed on the zero-th iteration and also for the printing of the residuals, unless the run comes to an untimely end (see the section on errors, page 57).

(6) Compute rectangular coordinates from

$$\vec{r}_c = \begin{pmatrix} x_c \\ y_c \\ z_c \end{pmatrix} , \tag{27}$$

where

$$\left.\begin{aligned} x_c &= r_c(\cos \ell \cos \Omega - \sin \ell \cos i \sin \Omega) \\ y_c &= r_c(\cos \ell \sin \Omega + \sin \ell \cos i \cos \Omega) \\ z_c &= r_c(\sin \ell \sin i) . \end{aligned}\right\} . \tag{28}$$

<u>Least Squares</u>

As its name implies, this program is concerned with orbit improvement with differential corrections. The position of the satellite (r) is a function of orbital elements, station coordinates, and time:

$$\vec{r} = \vec{r}\,(b,t) .$$

The approximate or observed position is $\vec{r}_0 = \vec{r}(b_0,t)$ .

Expansion about the approximate orbit $(b_0)$ is represented by

$$\vec{r}(b) = \vec{r}(b_0) + \frac{\partial \vec{r}}{\partial b}\, db .$$

Since the orbital elements are functions of the parameters p, then

$$\vec{r}(b) - \vec{r}(b_0) = \frac{\partial \vec{r}}{\partial b} \frac{\partial b}{\partial p}\, \Delta p .$$

What we solve for then is $\Delta p$, the corrections in the parameters needed to make the observed position $\vec{r}(b)$ correspond to the computed position $\vec{r}(b_0)$.

The expressions for $\vec{r} = \vec{r}(b,t)$ are extremely nonlinear. This fact and the need for rejecting poor observations require an iterative procedure.

The input elements are expressed as a sum of polynomials and sine terms. Any or all of the coefficients can be treated as variables, with the following restrictions:

(1) If the n-th degree terms are varied, then all the lower-order terms in the expression for that element must also be varied.

(2) There can be no more than 20 variables, including station coordinates. The variation codes are input with the orbit. (See the description of input, beginning on page 31, for more complete details on the variations allowed.)

Each observed quantity (e.g., a direction, such as altitude or azimuth) can be expressed as a linear equation:

$$\vec{r}_{obs.} - \vec{r}_{computed} = \sum_i \frac{\partial \vec{r}}{\partial \omega_i} + \sum \frac{\partial \vec{r}}{\partial \Omega_i} + \sum \frac{\partial \vec{r}}{\partial i_i} + \sum \frac{\partial \vec{r}}{\partial e_i}$$

$$+ \sum \frac{\partial \vec{r}}{\partial M_i} + \sum \frac{\partial \vec{r}}{\partial a_i} + \sum \frac{\partial \vec{r}}{\partial R_i} .$$

The term $\sum \frac{\partial \vec{r}}{\partial R_i}$ is the correction for station coordinates. These simultaneous equations are solved by the method of least squares.

The semimajor axis $a$ is computed from M and is included in the terms for M by the formulas that follow.

To the first order:

$$a = \left(\frac{k}{n^2}\right)^{1/3} = k^{1/3}\ n^{-2/3} \ ;$$

$$n = \frac{dM}{dt} = \dot{M} \text{ (polynomial part of M only) } ;$$

$$da = k^{1/3}\left(-\frac{2}{3}\right) n^{-5/3}\, dn\,,$$

$$= k^{1/3}\left(-\frac{2}{3}\right) \frac{1}{n^{5/3}}\, d\left(\frac{dM}{dt}\right)\;;$$

$$\vec{r}(b) - \vec{r}(b_0) = \frac{\partial \vec{r}}{\partial b}\Delta b, \text{ or, in terms of M and a,}$$

$$= \frac{\partial \vec{r}}{\partial M}\, dM + \frac{\partial \vec{r}}{\partial a}\, da,$$

$$= \frac{\partial \vec{r}}{\partial m}\, dM - a\,\frac{2}{3}\,\frac{dn}{n}\,\frac{\partial \vec{r}}{\partial a}\,.$$

We know that

$$\vec{r} = \vec{r}\; a(1 - e \cos E)$$

$$\frac{\partial \vec{r}}{\partial a} = \vec{r}\;(1 - e \cos E)\;;$$

therefore,

$$\vec{r}(b) - \vec{r}(b_0) = \frac{\partial \vec{r}}{\partial m}\, dM - \frac{2}{3}\,\frac{dn}{n}\, a\, \frac{\partial \vec{r}}{\partial a}$$

$$= \frac{\partial \vec{r}}{\partial m}\, dM - \frac{2}{3}\,\frac{\vec{r}}{n}\, dn\,.$$

The corrections to a are included in the time equation M.

<u>The development of equations of condition</u>.--We define the components of the vector $\vec{r}$, as $r_x$, $r_y$, $r_z$.

$r_x = x_c$, where $r_x$ is the geocentric inertial coordinate toward Aries;

$r_y = y_c$, where $r_y$ is such as to make a right-handed system; and

$r_z = z_c$, where $r_z$ is the geocentric inertial coordinate toward the pole.

The terms used in the linear equations (i.e., the equations of condition) are

$$\left.\begin{aligned}
\frac{\partial r_x}{\partial \omega_k} &= f_k\left[r_c(-\sin \ell \cos \Omega - \cos \ell \cos i \sin \Omega)\right], \\
\frac{\partial r_y}{\partial \omega_k} &= f_k\left[r_c(-\sin \ell \sin \Omega + \cos \ell \cos i \cos \Omega)\right], \\
\frac{\partial r_z}{\partial \omega_k} &= f_k\left[r_c \cos \ell \sin i\right], \\
\frac{\partial r_x}{\partial \Omega_k} &= f_k(-r_y), \\
\frac{\partial r_y}{\partial \Omega_k} &= f_k(r_x), \\
\frac{\partial r_z}{\partial \Omega_k} &= 0, \\
\frac{\partial r_x}{\partial i_k} &= f_k(r_z \sin \Omega), \\
\frac{\partial r_y}{\partial i_k} &= f_k(-r_z \cos \Omega), \\
\frac{\partial r_z}{\partial i_k} &= f_k[r_c \cos i \cos \ell], \\
\frac{\partial \vec{r}}{\partial e_k} &= f_k\left[\frac{\vec{r}_c}{r_c}\left(\frac{a e \sin^2 E}{1 - e \cos E} - a \cos E\right) + \frac{\partial \vec{r}}{\partial v}\left(\frac{\sin v}{1 - e \cos E} + \frac{\sin v}{1 - e^2}\right)\right], \\
\text{where} \quad \frac{\partial \vec{r}}{\partial v} &= \frac{\partial \vec{r}}{\partial \omega}; \\
\text{and} \quad \frac{\partial \vec{r}}{\partial M_k} &= f_k\left[\frac{\vec{r}_c}{r_c}\left(\frac{2\pi a e \sin E}{1 - e \cos E}\right) + \frac{\partial \vec{r}}{\partial v}\left(\frac{2\pi \sin v}{\sin E(1 - e \cos E)}\right)\right] + f_k^*\left[-\frac{2}{3}\frac{\vec{r}}{n}\right],
\end{aligned}\right\} \tag{29}$$

where the $f_k$, $f_k^*$ are functions of the definition of the input elements.

$$\frac{\partial \vec{r}_c}{\partial R} = \begin{pmatrix} -\cos \theta & \sin \theta & 0 \\ -\sin \theta & -\cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

The following chart shows the relationship between the variables and the values of $f_k$ and $f_k^*$ :

| Variable | $f_k$ | $f_k^*$ (see footnote 1) |
|---|---|---|
| $P$ | $1$ | $0$ |
| $P_1$ | $t$ | $1$ |
| $P_2$ | $t^2$ | $2t$ |
| . | . | . |
| . | . | . |
| . | . | . |
| $P_n$ | $t^n$ | $nt^{n-1}$ |
| $S_0$ | $\sin(S_1+S_2t)$ | - |
| $S_1$ | $S_0 \cos(S_1+S_2t)$ | - |
| $S_2$ | $S_0t \cos(S_1+S_2t)$ | - |
| $H_0$ | $\exp[H_2 \ell n(H_1-T)]$ | - (see footnote 2) |
| $H_1$ | $\frac{H_2}{H_1 - T} \exp[H_2 \ell n(H_1-T)]$ | - |
| $H_2$ | $\ell n(H_1-T)\ H_0 \exp[H_2 \ell n(H_1-T)]$ | - |

[1] The $f_k^*$ terms are for the term in da (see page 17). Since n and hence a are defined as the polynomial part of $\dot{M}$, the sine and exponential terms are omitted.

[2] where $T = \tau + t$ (i.e., the time in MJD) and $\tau$ is the epoch.

As described, the equations of condition are expressed in terms of positions. To determine the residual ($\vec{r}_{obs.} - \vec{r}_{computed}$), computed values are used for quantities not observed. In other words, if only the direction is observed, then the computed range is used to compute $\vec{r}_{obs.}$ Similarly, if only the range is observed, then the computed direction is used. This has the twofold advantage of strengthening the solution and allowing a uniform treatment of the equations of condition.

To avoid correlations introduced by using rectangular coordinates, the resulting 3 x n matrix (n unknowns) is multiplied by a matrix that rotates the coefficients to the coordinate system of the observation. In the process, the number of residuals is reduced to the number originally observed--3 for position, 2 for direction, and 1 for range.

## Station Position

If $\vec{R}$ is the station position in the earth-centered geodetic system (i.e., the station coordinates are constants in this system), and $\vec{R}_s$ is the station position in the coordinate system of the satellite (the inertial geocentric system), then the relation between these $\vec{R}$, $\vec{R}_s$ systems is

$$\vec{R}_s = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \vec{R} , \tag{30}$$

where θ is the sidereal angle.

Station coordinate corrections ($\Delta\vec{R}$) are computed in the geocentric geodetic system. Since we require the station position ($\vec{R}'_s$) in the inertial geocentric system, the station position used is computed from

$$\vec{R}'_s = \vec{R}_s + \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \Delta R . \tag{31}$$

Residual computation.--With $\Delta\vec{r} = \Delta\vec{\rho}$ (observed position-computed position) as the residual vector, the computed position, $\vec{\rho}_c$, is determined from the equation

$$\vec{\rho}_c = \vec{r}_c - \vec{R}'_s . \tag{32}$$

If the observation is a position measurement (i.e., if there are three observed quantities), then

$$\vec{\Delta r} = \vec{\rho}_{obs.} - \vec{\rho}_c \; . \tag{33}$$

If the direction is observed (i.e., if there are two observed quantities), then

$$\vec{\Delta r} = \rho_c \hat{\rho}_{obs.} - \vec{\rho}_c \; . \tag{34}$$

If only the range is observed (i.e., if there is one observed quantity), then

$$\vec{\Delta r} = \frac{\rho_{obs.} - \rho_c}{\rho_c} \hat{\rho}_c \; . \tag{35}$$

As I have previously mentioned, the equation of condition, $\frac{\partial x}{\partial b} \vec{db}$, and the residuals, $\vec{dr}$, are rotated into the coordinate system of the observation by multiplying both sides of the equation by the matrix C:

$$C \frac{\partial x}{\partial b} \vec{db} = C \vec{dr} \; .$$

The values of $C \vec{dr}$ are printed as the residuals. This matrix contains the weighting factor (which is taken out of the residual when it is printed).

<u>The $C_{\alpha\delta}$ matrix for $\alpha$ and $\delta$ observations</u>.--The observed quantities $\alpha$ and $\delta$ define the coordinate system of the residuals. It is important to realize that these are the observed $\alpha$, $\delta$, but corrections have been added to bring them into the defined sidereal system.

In principle the C matrix, with weighting, is

$$C_{\alpha\delta} = \frac{1}{\sigma_i \rho_c} \begin{pmatrix} -\cos\alpha \sin\delta & -\sin\alpha \sin\delta & \cos\delta \\ -\sin\alpha & \cos\alpha & 0 \\ \rho_x & \rho_y & \rho_z \end{pmatrix} , \tag{36}$$

where $\sigma_i$ represents the uncertainty of this observation, that is to say,

$$\frac{1}{\sigma_i}\begin{pmatrix} d\delta \\ \cos\,\delta\,\,d\alpha \\ \frac{dr^*}{r} \end{pmatrix} = C_{\alpha\delta}\,\overrightarrow{dr} = C_{\alpha\delta}\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} . \tag{37}$$

To make computation simpler, $C_{\alpha\delta}$ is generally derived by using the direction cosines

$$\vec{\rho}_o = \begin{pmatrix} x \\ y \\ z \end{pmatrix} . \tag{38}$$

If $k = \sqrt{x^2+y^2} = r\,\cos\,\delta'$, where the prime denotes reduced observation, then

$$C_{\alpha,\delta} = \frac{1}{\sigma_i\rho_c}\begin{pmatrix} -\frac{zx}{rk} & -\frac{zy}{rk} & \frac{k}{r} \\ -\frac{y}{k} & \frac{x}{k} & 0 \\ \frac{x}{r} & \frac{y}{r} & \frac{z}{r} \end{pmatrix} . \tag{39}$$

*if range is observed.

If altitude (a) and azimuth (Az) are observed,

$$C_{a,Az} = \frac{1}{\sigma_i \rho_c} \begin{bmatrix} \begin{pmatrix} -\sin Az \sin a & -\cos Az \sin a & \cos a \\ \cos Az & \sin Az & 0 \end{pmatrix} \begin{pmatrix} -\sin(\lambda+\theta) & -\cos(\lambda+\theta)\sin\phi & \sin(\lambda+\theta)\cos\phi \\ \cos(\lambda+\theta) & -\sin(\lambda+\theta)\sin\phi & \sin(\lambda+\theta)\cos\phi \\ 0 & \cos\phi & \sin\phi \end{pmatrix} \\ \begin{matrix} \dfrac{\rho_x}{\rho_0} & \dfrac{\rho_y}{\rho_0} & \dfrac{\rho_z}{\rho_0} \end{matrix} \end{bmatrix}^{\dagger} . \tag{40}$$

The altitude-azimuth used in this rotation matrix must be the original a-Az, with the corrections for refraction added before the A matrix (see page 7) rotation is applied. The program saves these quantities for use in the C matrix. Here again the $\sigma_i$ represents the weighting of this observation.

$$C_{a,Az} = \frac{1}{\sigma_i \rho_a} \begin{bmatrix} \begin{pmatrix} -\sin Az \sin a & -\cos Az \sin a & \cos a \\ \cos Az & \sin Az & 0 \end{pmatrix} \begin{pmatrix} -\sin(\lambda+\theta) & \cos(\lambda+\theta) & 0 \\ -\cos(\lambda+\theta)\sin\phi & -\sin(\lambda+\theta)\sin\phi & \cos\phi \\ \sin(\lambda+\theta)\cos\phi & \sin(\lambda+\theta)\cos\phi & \sin\phi \end{pmatrix} \\ \begin{matrix} \dfrac{\rho_x}{\rho_0} & \dfrac{\rho_y}{\rho_0} & \dfrac{\rho_z}{\rho_0} \end{matrix} \end{bmatrix} . \tag{41}$$

$$\frac{1}{\sigma_i} \begin{pmatrix} da \\ -\cos a \; dAz \\ \dfrac{dr^*}{r} \end{pmatrix} = C_{a,Az} \, \vec{dr} = C_{a,Az} \begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} . \tag{42}$$

† The second matrix is the transpose of the A matrix defined in equation (11).

* if range is observed.

If $\ell$,m (direction cosines) are observed:

$$C_{\ell m} = \frac{1}{\sigma_i \rho_c} \begin{bmatrix} -\sin(\lambda+\theta) & \cos(\lambda+\theta) & 0 \\ -\cos(\lambda+\theta)\sin\Phi & -\sin(\lambda+\theta)\sin\Phi & \cos\Phi \\ \frac{\rho_x}{\rho_0} & \frac{\vec{\rho}_y}{\rho_0} & \frac{\vec{\rho}_z}{\rho_0} \end{bmatrix} \tag{43}$$

$$\frac{1}{\sigma_i}\begin{pmatrix} \ell \\ m \\ n \end{pmatrix} = C_{\ell M}\,\overrightarrow{dr} = C_{\ell M}\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} \quad . \tag{44}$$

If range $(\rho_0)$ is observed

$$C_{\ell m} = \frac{1}{\sigma_i \rho_c}\,(\rho_x\ \rho_y\ \rho_z) \ . \tag{45}$$

Each observation (submatrix) is weighted with the assumed accuracy of the observation $(\sigma_i)$,

$$C(\frac{1}{\sigma_i})\ \overrightarrow{dr} = C(\frac{1}{\sigma_i})\ \frac{\partial x}{\partial b}\ \overrightarrow{db} \quad . \tag{46}$$

Note that $\sigma_i$ is independently determined and is not the outcome of an internal calculation. Each observed quantity receives the same weight; weighting becomes important when several different kinds of observations are combined.

This is, however, not true statistical weighting as in, for instance, radar observations, where the range can be determined more accurately than the direction. A true weighting gives proportionate weight to the different types of observations, but when we combine various types of observations, the less accurate ones dominate. If, however, the weights are assigned so as to insure equal effects on the orbit, the weight ceases to be an indication of the assumed accuracy of the instrument.

Note that, except in the case of photoreduced observations, the weight is input with the identification of the observing station. For photoreduced observations (indicated by an observation number in the 70000 series) we use the weight punched on the observation card.

The least-squares solution normally assumes statistical errors in the observed position as well as errors in the observed time, which are often larger than those in position. The program has two mechanisms for combating these errors. First is the option for rotated residuals, which does two least-squares solutions in terms of du and dw after a successful orbit improvement (see REITER control card, page 48). The program computes the relative weight on the first iteration and applies it during the second. The alternate method involves the multiplication of the equations-of-condition submatrix of each observation by a weight matrix.

If, as an example of the second option, the problem is stated

$$\frac{\partial \vec{r}}{\partial b} \frac{\partial b}{\partial p} \Delta p = \Delta \vec{\rho} ,$$

where each quantity is a matrix, we multiply the left-hand side by W, a weight matrix (Veis, 1960):

$$W = \frac{1}{\sigma_i^2} \begin{pmatrix} \cos^2 \varphi + \frac{1}{k^2} \sin^2 \chi & \left(1 - \frac{1}{k^2}\right) \sin \chi \cos \chi \\ 1 - \frac{1}{k^2} \sin \psi \cos \chi & \sin^2 \psi + \frac{1}{k^2} \cos^2 \chi \end{pmatrix} , \tag{47}$$

where $\sigma_i$ is the observational uncertainty; $\sigma_t$ is the uncertainty in time; $\rho^2$ is a plate constant (= 1.0); $k^2$ is $\rho^2 + \chi^2 \frac{\sigma_t^2}{\sigma^2}$ ; and $\chi$ is the position angle.

## Rotated Residuals

The first method of overcoming timing errors in the observations, as we have indicated, is to perform the least-squares solution in terms of the residuals du and dw (the residuals in and perpendicular to the plane of the orbit). The development and rationale of this method are documented in Izsak (1961). The program is set up to handle only observations with two observed quantities; it will not work on runs requiring the determination of station coordinates.

We define (see figure 5)

$$\vec{e}_N = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \tag{48}$$

$$\vec{e}_A = \begin{pmatrix} \cos\delta \quad \cos(\alpha-\Omega) \\ \sin i \sin\delta + \cos i \cos\delta \quad \sin(\alpha-\Omega) \\ \cos i \sin\delta - \sin i \cos\delta \quad \sin(\alpha-\Omega) \end{pmatrix}, \tag{49}$$

$$\frac{\vec{r}}{a} = \frac{r}{a} \begin{pmatrix} \cos u \\ \sin u \\ 0 \end{pmatrix}, \tag{50}$$

where $u = v + \omega$;

$$\frac{\vec{r}}{n} = \frac{a}{\sqrt{1-e^2}} \begin{pmatrix} -\sin(u+\eta) \\ \cos(u+\epsilon) \\ 0 \end{pmatrix} \tag{51}$$

where $\eta = e \sin\omega$ and $\epsilon = e \cos\omega$;

$$\vec{e}_B = \begin{pmatrix} -\sin(\alpha-\Omega) \\ \cos i \quad \cos(\alpha-\Omega) \\ -\sin i \quad \cos(\alpha-\Omega) \end{pmatrix}, \tag{52}$$

$$\vec{e}_c = \begin{pmatrix} -\sin\delta \cos(\alpha-\Omega) \\ \sin i \cos\delta - \cos i \sin\delta \quad \sin(\alpha-\Omega) \\ \cos i \cos\delta + \sin i \sin\delta \quad \sin(\alpha-\Omega) \end{pmatrix}, \tag{53}$$

$$\vec{e}_z = \begin{pmatrix} 0 \\ \sin i \\ \cos i \end{pmatrix}. \tag{54}$$

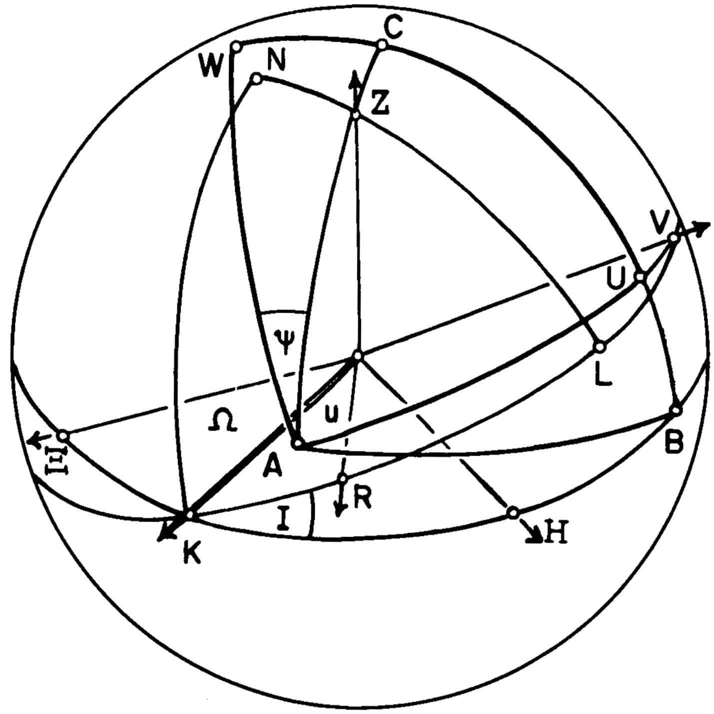



Figure 5.--The geometry of rotated residuals. Ξ, H, Z is the sidereal coordinate system; the points R and V indicate the directions of the geocentric position and velocity of the satellite. (Published earlier as Figure 1 in Smithsonian Astrophysical Observatory Special Report 73 [Izsak, 1961b].)

The components of the equations of condition for dw are

$$\left.\begin{aligned}
\frac{\partial r}{\partial \omega_k} &= f_k \frac{1}{\rho}\left[\vec{e}_w \cdot (\vec{e}_N \times \vec{r})\right] ; \\
\frac{\partial r}{\partial \Omega_k} &= f_k \frac{1}{\rho}\left[\vec{e}_w \cdot (\vec{e}_z \times \vec{r})\right] ; \\
\frac{\partial r}{\partial i_k} &= f_k \frac{r}{\rho} \sin (v+\omega) \, \vec{e}_w \cdot \vec{e}_n ; \\
\frac{\partial r}{\partial e_k} &= f_k \frac{1}{\rho} \vec{e}_w \cdot \left\{ (\vec{e}_N \times \vec{r}) \frac{a/r}{\sqrt{1-e^2}} \sin E - \vec{a} \right\} ; \\
\frac{\partial r}{\partial M_k} &= 0 - f_k^* \frac{2a}{3n} \frac{1}{\rho} (\vec{e}_w \cdot \vec{r}/a) \quad .
\end{aligned}\right\} \qquad (55)$$

The components of the equations of condition for du are

$$\left.\begin{aligned}
\frac{\partial r}{\partial \omega_k} &= wtf_k \frac{1}{\rho}\left[\vec{e}_u \cdot (\vec{e}_N \times \vec{r})\right] ; \\
\frac{\partial r}{\partial \Omega_k} &= wt \; f_k \frac{1}{\rho}\left[\vec{e}_u \cdot (\vec{e}_z \times \vec{r})\right] ; \\
\frac{\partial r}{\partial i_k} &= wt \; f_k \frac{r}{\rho} \sin (v+\omega) \, \vec{e}_u \cdot \vec{e}_n ; \\
\frac{\partial r}{\partial e_k} &= wt \; f_k \frac{1}{\rho} \vec{e}_u \cdot \left\{ (\vec{e}_N \times \vec{r}) \frac{a/r}{\sqrt{1-e^2}} \sin E - \vec{a} \right\} ; \\
\frac{\partial r}{\partial M_k} &= wt \; f_k \frac{2\pi}{\rho} (\vec{e}_u \cdot \vec{r}/n) - f_k^* \frac{2}{3} \frac{a}{n} (\vec{e}_u \cdot \vec{r}/a) ; \\
\vec{e}_w &= \frac{\vec{e}_A \times \frac{\vec{r}}{n}}{\left|\vec{e}_A \times \frac{\vec{r}}{n}\right|} ; \\
\vec{e}_u &= \vec{e}_w \times \vec{e}_A \quad .
\end{aligned}\right\} \qquad (56)$$

The residual for dw is computed by

$$dw = - \sin \psi \cos \delta \, d\alpha + \cos \psi \, d\delta$$
$$du = wt \, (\cos \psi \cos \delta\alpha + \sin \varphi \, d\delta) \,, \tag{57}$$

where

$$\cos \psi = \vec{e}_w \cdot \vec{e}_c \,,$$

and

$$\sin \psi = - \vec{e}_w \cdot \vec{e}_s \,,$$

where $\psi$ is the position angle, and wt is the weight assigned to the residual in dw. On the first iteration this weight is 1.0; on the second it is computed from the residuals of the first by

$$wt = \sqrt{\frac{\sum dw^2}{\sum du^2}} \,. \tag{58}$$

The program prints the value of this weight.

The program functions in essentially the same way with rotated residuals as it does with orbit improvement in $\cos \delta \, d\alpha$, $d\delta$. The process starts with an improvement in $\cos \delta \, d\alpha$, $d\delta$ until normal convergence, then prints the improved orbit and residuals. Next it performs two more iterations, deleting the C matrix multiplications and using the new equations of condition. On the first iteration the program computes and prints the relative weight of du, dw. In the second, it applies this weight. This weight is in addition to the weighting by the assumed accuracy ($\sigma_i$).

We would normally use this processing mode only for final evaluation of high-precision observations. The orbits obtained in this way are not substantially better than those derived by the usual method, and this program is more time-consuming, because the expressions for the equations of conditions are more complex. The residuals, however, are much more useful for evaluation, as they are separated into components with no timing errors (du) and those with timing errors (dw).

Both these options are set up for observations with 2 observed quantities; the results are meaningful only for precisely reduced Baker-Nunn observations.

Today time measurements for other observations are much better than positional measurements. It makes sense to weight observed quantities only when the accuracy of their determination is roughly equivalent.

## Rejection Criteria

An observation is not included if

$$\frac{1}{\sigma_i}|\overrightarrow{dr}| > 3\sigma ,$$

where $\sigma$ is the over-all standard error of the previous iteration. This rejection criterion seems quite effective. The zero-th iteration assumes a $\sigma$ of $1 \times 10^6$, which includes all but the observations greatly in error.

The least-squares iteration is assumed to converge if the change of $\sigma$ on two successive iterations differs by less than 1 part in 100, or if

$$\frac{\sigma_i - \sigma_{i-1}}{\sigma_i} < .01 .$$

## Method of Solution

Each equation of condition is conceptually appended to a matrix. The solution to this system lies in the following:

If A is the $i \times n$ matrix of coefficients (n unknowns and i observed quantities) db is the desired column vector, and the dr is the O-C (observed minus computed positions) or residuals.

$$\underset{(i \times n)}{A} \ \underset{(n \times 1)}{\overline{db}} = \underset{(n \times 1)}{\overline{dr}}$$

$$A^T_{n \times i} A_{i \times n} db_{n \times 1} = A^T_{n \times i} dr_{i \times 1}$$

$$M_{n \times n} db_{n \times 1} = b_{n \times 1}$$

$$db_{n \times 1} = M^{-1}_{n \times n} b_{n \times 1} .$$

# Input

The usefulness of a program like the DØI depends largely on the flexibility and generality of the input and on the user's ability to make use of the various options. This section on input provides an index for the control cards as well as a description of the usefulness of the options.

## Formats

Except for the observation cards, all input is free-field. For a complete description of the free-field input conventions as well as the input-output techniques, see Gaposchkin (1963). Suffice it to say here that in the input-output program the fields are delimited by one (or more) blanks and that BCD information, integers, and floating-point numbers (which must contain a decimal point) are recognized by their syntax. The order of items (if more than one is needed) is therefore important. This applies to BCD control information as well.

Element definition cards.--These cards contain four types of information, in this order:

(1) satellite name;
(2) the combination of polynomials, sine terms, and hyperbolic terms that make up the 5 elements;
(3) the numerical coefficients to be improved (i.e., treated as unknowns); and
(4) the numerical values for the elements.

The first field contains the year of launch; the second, the satellite name, which may be either the Greek letter (spelled out) or the launch number (the COSPAR code). If the Greek-letter designation is used and double-letter names are needed, then the third field includes the second Greek letter. In the next field is the particle number or, in the case of the COSPAR code, the letter designating the particle number.

Examples:

| Satellite | Punched Designation |
|---|---|
| 1961 $\alpha$1 | 1961 ALPHA 1 or 1961 1 A |
| 1961 $\alpha\beta$2 | 1961 ALPHA BETA 2 or 1961 26 B |

Satellite identification from the elements is checked against the satellite number on the observations. If they do not agree, the observations cannot be identified and are therefore rejected. The first card, with any title information, is reproduced on the output listing.

The next fields define the form of the equations defining the five elements and indicate which coefficients will be treated as unknowns. Each element must have a polynomial (0 to 7th order); several have one or more sine terms and one or more hyperbolic terms added. The independent variable is time from epoch. The polynomial definition of an element, P, must come first, followed immediately by the degree n of the polynomial, then by n + 1 digits, either 0 or 1, depending on whether the corresponding coefficient of the polynomial is to be constant or improved. A third-degree polynomial with constant and linear terms as variables would thus be P31100. Sine terms, which always have three coefficients, are of the form

$$S_0 \sin (S_1 + S_2 t);$$

the variation of any of these coefficients is expressed as a digit following the S designation; for example, S100 means vary the $S_0$ term. The hyperbolic terms are of the form

$$H_0 \exp (H_2 \ln (H_1 - (T + t))) .$$

The variation of any coefficient is shown by a digit following the H designation, where T is the epoch (e.g., H100).

The elements are defined in the following order:

$\omega$ = argument of perigee;
$\Omega$ = argument of the ascending node;
i = inclination;
e = eccentricity;
M = mean anomaly.

For example, if

$$\omega = \omega_0 + \omega_1 t + \omega_2 \sin (\omega_3 + \omega_4 t) ,$$

$$\Omega = \Omega_0 + \Omega_1 t + \Omega_2 t^2 ,$$

$$i = i_0 ,$$

$$e = e_0 + e_1 t ,$$

$$M = M_0 + M_1 t + M_2 t^2 + M_3 \sin (M_4 + M_5 t) ,$$

and all the constant terms and the amplitudes of the sine terms are to be varied, then the code would be

P110 S100 P2100 P01 P110 P2100 S100 .

Following this definition are the epoch (double precision, i.e., two numbers) and the coefficients, appearing in the order implied by the definition. Even when a term equals zero, it must still be punched.

The previous orbit, when punched, would look like this:

```
120.   5.2
.15   10.205631   .01562   .152E-3
.025          1.635E-4      72.1   .1   .5E-5
37000. .0   100.    .1  .01  120.   3.5   50.
P110   S100   P2100    P01   P110   P2100   S100
1961 ALPHA 1          TEST CASE
```

All the coefficients must have decimal points. The exponential notation (e.g., 1.E-5) is frequently used here. As many cards as necessary may be used, with as many or as few fields per card as desired, provided that there is at least one blank between successive fields. Some information must precede column 12 on each card, or the card will be treated as a blank. Under no condition may fields continue from one card to another.

The units of the coefficients are

$\omega$, $\Omega$, i  degrees,

e  dimensionless,

M  revolutions,

with the independent variable time t in days.

<u>Epoch Cards</u>

Use of the epoch card changes epoch of the orbit to a new epoch. Simultaneously, the orbit being computed in the machine at the time is updated. This change is essentially a shift in the origin. The epoch card has two numbers, the integer and fractional parts of the new epoch.

Example:

```
37000.   0.
```

More sophisticated use of the epoch card involves computing orbits for successive epochs. This procedure can give consecutive orbits for long periods of time. This epoch card has four numbers: the integer part of the initial epoch, the fractional part, the amount of time step, and the interval of time to be considered with each epoch.

Example:

```
37000.   0.   2.   4.
```

In this example the epoch of the first orbit is to be 37000.0, using the observations from 36998.0 to 37002.0. The next orbit is 37001.0, using the observations from 36999.0 to 37003.0, and so on. The program assumes that the observations are chronological and continues to compute them until the input file is exhausted.

Observation Cards

The observation-card format is fixed. By January 1, 1964, all the observation cards at SAO were in the format represented in figure 6. Columns 1-12 are the identification, which consists of two fields.

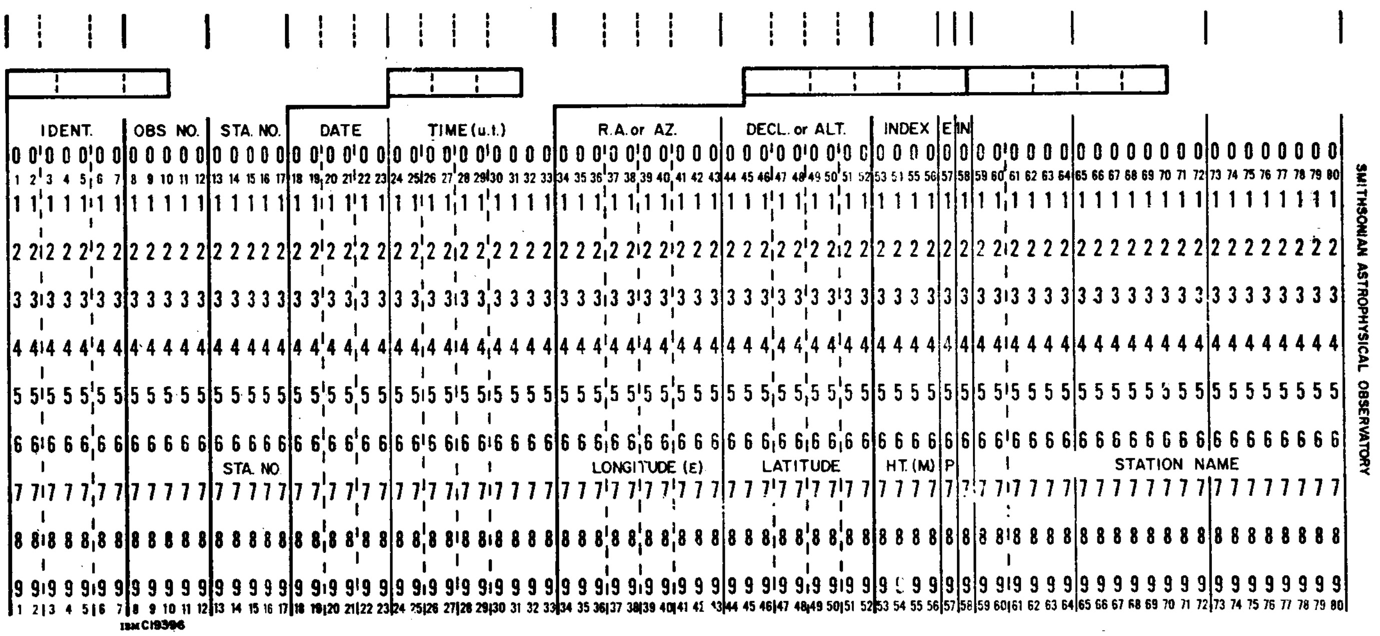



Figure 6.--Sample observation card.

Columns 1-7 contain the satellite designation, which remains the same for all observations of the same object:

cols. 1-2, the year of launch, from 1900;
cols. 3-5, the number of the launch in that year;
cols. 6-7, the particle number.

Satellite 1959 $\alpha 1$, for example, would be designated 5900101.

Columns 8-12 contain the observation number. Each observation of a satellite in a given year is designated by a different number. A numbering scheme aids in identifying different sources of observations:

| | |
|---|---|
| 1- 9999 | Catch-all |
| 10000-19999 | Baker-Nunn, field-reduced |
| 20000-29999<br>30000-39999<br>40000-49999 | Moonwatch |
| 50000-59999<br>60000-69999 | Miscellaneous observations |
| 70000-79999 | Baker-Nunn, precisely reduced |
| 80000-89999<br>90000-99999 | Minitrack |

SAO uses a (-) punch in column 1 to indicate that an observation has been published.

The satellite identification on each observation card and the identification of the orbit are compared; if the two do not agree, the observation is rejected. This fact will not be useful for selective use of observations, as the program will not proceed automatically to the next.

Columns 13-17 contain the station number.

Columns 18-23 contain the date of the observation:

cols. 18-19, year, from 1900;
cols. 20-21, month;
cols. 22-23, day;

An observation made on May 29, 1961, for example, would be indicated by 610529.

Columns 24-33 contain the time designation. Different types of observations have different time systems, which are not based on local time. The systems for each type are

Field-reduced Baker-Nunn--WWV received;
Photoreduced Baker-Nunn--A1;
Minitrack--WWV emitted;
Some field-reduced--WWV emitted;
Moonwatch--WWV received.

cols. 24-25, hour;
cols. 26-27, minute;
cols. 28-29, seconds;
cols. 30-33, fractions of seconds, to .1 millisecond.

Column 56 contains a code that indicates the type of observation (see page 38). The interpretation of the following fields depends on column 56.

If column 56 is 0, then the observation is right ascension, declination ($\alpha$, $\delta$), and

| | |
|---|---|
| col. 34, | blank; |
| cols. 35-36, | hours of $\alpha$; |
| cols. 37-38, | minutes of $\alpha$; |
| cols. 39-40, | seconds of $\alpha$; |
| cols. 41-43, | fraction of seconds to .001 second; |
| col. 44, | sign of $\delta$; |
| cols. 45-46, | degrees of $\delta$; |
| cols. 47-48, | minutes of $\delta$; |
| cols. 49-50, | seconds of $\delta$; |
| cols. 51-52, | fractions of seconds to .01 second. |

If column 56 is 1 or 3, then the observation is altitude and azimuth, corrected or uncorrected for refraction, and

| | |
|---|---|
| cols. 34-36, | degrees of azimuth; |
| cols. 37-38, | minutes of azimuth; |
| cols. 39-40, | seconds of azimuth; |
| cols. 41-43, | fractions of seconds to .001 second; |
| col. 44, | blank; |
| cols. 45-46, | degrees of altitude; |
| cols. 47-48, | minutes of altitude; |
| cols. 49-50, | seconds of altitude; |
| cols. 51-52, | fractions of seconds to .01 second. |

If column 56 is 4 or 5, then the observation is direction cosines ($\ell$,m), corrected or uncorrected for refraction, and,

| | |
|---|---|
| col. 34, | sign of l (blank or minus); |
| cols. 35-42, | l to 8 decimal places (decimal point implied before column 35); |
| col. 43, | blank; |
| col. 44, | sign of m (blank or minus); |
| cols. 45-52, | m to 8 decimal places (decimal point implied before column 45); $n = \sqrt{\ell^2 + m^2}$. |

If column 56 is 6, then the observation is range, and

| | |
|---|---|
| col. 34, | blank; |
| cols. 35-42, | in megameters (decimal point implied before column 37 allows range observations to be specified to $10^{-6}$ megameter or 1 meter). |

Column 53 contains the time-precision index defined as follows (Veis, 1959):

| Code Number | Standard Error in Timing ($\sigma_t$) |
|---|---|
| 0 | No estimate |
| 1 | $\sigma_t \leq .0003^{sec}$ |
| 2 | $.0003 < \sigma_t \leq .002$ |
| 3 | $.002 < \sigma_t \leq .005$ |
| 4 | $.005 < \sigma_t \leq .02$ |
| 5 | $.02 < \sigma_t \leq .05$ |
| 6 | $.05 < \sigma_t \leq .2$ |
| 7 | $.2 < \sigma_t \leq .5$ |
| 8 | $.5 < \sigma_t \leq 2.0$ |
| 9 | $\sigma_t > 2.0$ |

This code is not normally used, but it is referred to under the USEWMX control option.

Columns 54-55 contain the position precision index, defined as follows (from Veis, 1959):

| Code number | Standard error in direction ($\sigma_D$) | Code number | Standard error in direction ($\sigma_D$) |
|---|---|---|---|
| 00 | No estimate | 07 | $6''.5 < \sigma_D \leq 7''.5$ |
| 01 | $\sigma_D \leq 1''.5$ | 08 | $7''.5 < \sigma_D \leq 8''.5$ |
| 02 | $1''.5 < \sigma_D \leq 2''.5$ | 09 | $8''.5 < \sigma_D \leq 9''.5$ |
| 03 | $2''.5 < \sigma_D \leq 3''.5$ | 10 | $9''.5 < \sigma_D \leq 10''.5$ |
| 04 | $3''.5 < \sigma_D \leq 4''.5$ | 11 | $10''.5 < \sigma_D \leq 11''.5$ |
| 05 | $4''.5 < \sigma_D \leq 5''.5$ | 12 | $11''.5 < \sigma_D \leq 12''.5$ |
| 06 | $5''.5 < \sigma_D \leq 6''.5$ | 13 | $12''.5 < \sigma_D \leq 13''.5$ |

| Code number | Standard error in direction ($\sigma_D$) | Code number | Standard error in direction ($\sigma_D$) |
|---|---|---|---|
| 14 | $13''.5 < \sigma_D \le 14''.5$ | 32 | $1'.7 < \sigma_D \le 2'.1$ |
| 15 | $14''.5 < \sigma_D \le 15''.5$ | 33 | $2'.1 < \sigma_D \le 2'.7$ |
| 16 | $15''.5 < \sigma_D \le 16''.5$ | 34 | $2'.7 < \sigma_D \le 3'.5$ |
| 17 | $16''.5 < \sigma_D \le 17''.5$ | 35 | $3'.5 < \sigma_D \le 4'.4$ |
| 18 | $17''.5 < \sigma_D \le 18''.5$ | 36 | $4'.4 < \sigma_D \le 5'.8$ |
| 19 | $18''.5 < \sigma_D \le 19''.5$ | 37 | $5'.8 < \sigma_D \le 7'.5$ |
| 20 | $19''.5 < \sigma_D \le 20''.5$ | 38 | $7'.5 < \sigma_D \le 9'.7$ |
| 21 | $20''.5 < \sigma_D \le 22''$ | 39 | $9'.7 < \sigma_D \le 13'$ |
| 22 | $22'' < \sigma_D \le 23''.5$ | 40 | $13' < \sigma_D \le 17'$ |
| 23 | $23''.5 < \sigma_D \le 26''$ | 41 | $17' < \sigma_D \le 22'$ |
| 24 | $26'' < \sigma_D \le 29''$ | 42 | $22' < \sigma_D \le 28'$ |
| 25 | $29'' < \sigma_D \le 33''$ | 43 | $28' < \sigma_D \le 37'$ |
| 26 | $33'' < \sigma_D \le 38''$ | 44 | $37' < \sigma_D \le 49'$ |
| 27 | $38'' < \sigma_D \le 45''$ | 45 | $49' < \sigma_D \le 1^\circ.1$ |
| 28 | $45'' < \sigma_D \le 54''$ | 46 | $1^\circ.1 < \sigma_D \le 1^\circ.4$ |
| 29 | $54'' < \sigma_D \le 1'.1$ | 47 | $1^\circ.4 < \sigma_D \le 1^\circ.8$ |
| 30 | $1'.1 < \sigma_D \le 1'.3$ | 48 | $1^\circ.8 < \sigma_D \le 2^\circ.4$ |
| 31 | $1'.3 < \sigma_D \le 1'.7$ | 49 | $2^\circ.4 < \sigma_D$ |

This precision index is used only with photoreduced observations. All other observations use the precision input with the station coordinate cards (see BINSTA control card, page 41).

Column 56, as we have seen, contains the index of observation types, defined as follows:

| Index | Explanations |
|---|---|
| 0 | Right ascension, declination |
| 1 | Altitude, azimuth (corrected for refraction) |
| 2 | Illegal |
| 3 | Altitude, azimuth (uncorrected for refraction) |
| 4 | $\ell$,m (direction cosines, corrected for refraction) |
| 5 | $\ell$,m (direction cosines, uncorrected for refraction) |
| 6 | Range (megameters) |

Column 57 contains an index referring to the date of equator and equinox to which the observation is referred (meaningful for right ascension, declination only).

| Index | Date |
|---|---|
| 0 | Date of observation |
| 1 | 1855.0 |
| 2 | 1875.0 |
| 3 | 1900.0 |
| 4 | 1950.0 |

Column 58 is the instrument description (ignored by the program):

| Code number | Optical observations | Code number | Electronic observations |
|---|---|---|---|
| 0 | Naked eye and binoculars, visual. | 0 | Minitrack Mark 1. |
| 1 | Standard Moonwatch telescope, visual. | 1 | Minitrack Mark 2. |
| 2 | Apogee telescope, astronomical refractor or reflector, theodolite, visual. | 2 | Interferometer observations from radio observatories |
| 3 | Baker-Nunn camera, photographic. | 3 | Doppler observations from radio observatories. |
| 4 | Small missile telecamera, tracking cameras with focal length 20 inches or greater, photographic. | 4 | Microlock. |
| 5 | Cinetheodolite, tracking cameras with focal length less than 20 inches, photographic. | 5 | Doppler observations from communications systems. |
| 6 | Harvard meteor camera (Super-Schmidt), photographic. | 6 | Doppler observations from missile ranges. |
| 7 | Stationary telescope or camera with focal length equal to or less than 10 inches, photographic. | 7 | Radar. |
| 8 | Stationary telescope or camera with focal length greater than 10 inches, photographic. | 8 | Unused digit. |
| 9 | Other instruments, or instrument unknown. | 9 | Miscellaneous. |

Columns 59-64 indicate range and direction. If an observation includes both a range and a direction, the direction is punched with the appropriate code in column 56 and the range is punched in columns 59-64. If a range is not available, these columns must be blank.

cols. 59-63, range in kilometers (decimal point implied before column 60);
col. 64, power of 10 multiplying the range.

In the least-squares solution all observed quantities from the same observation receive the same weight. Note, however, that in the case of radar observations, for instance, this is not necessarily true. Be careful.

Columns 65-71 contain the conversion from the UT2 to the A1 time system, i.e., A1-UT2

col. 65, sign;
cols. 66-67, tens and units digits of seconds (maximum correction, 99 seconds);
cols. 68-71, fractions of seconds to .1 millisecond.

Columns 73-78 contain alpha numeric information identifying the station or observing site.

N.B.: Control cards and observations may not be intermixed in the input deck.

<u>Control Cards</u>

The following control cards are identified by the first alpha numeric field on the card. This field must start before column 12. The free-field conventions hold (see page 31).

1. The asterisk comment card

```
* OUTPUT TAPE. . . .
* PRINTED ON-LINE AND ON TAPE
* COMMENT CARD
```

2. The USE card

Only the observations from the stations punched on this card will be used in the following run. If more than one card is needed to list the station numbers, each successive card should also have USE punched on it. This card, operative for the next satellite only, must precede the observations.

```
USE    9004    9005    9006
USE    9001    9002    9003
```

3. The ØMIT card

Observations from stations punched on this card will be omitted in the following run. If more than one card is needed to list the station numbers, each additional card should have ØMIT punched on it. This card, operative for the next satellite only, must, like the USE card, precede the observations.

```
ØMIT    9010    9011    9012
```

4. The BINSTA card

The DØI uses a binary station tape for input of the station coordinates. Generally the tape is on B5 (see IØTAPE, page 42); in normal circumstances it is prepared ahead of time and mounted at running time. The BINSTA card instructs the program to write a binary station tape. The station coordinates cards, which follow the BINSTA card, are terminated by a blank card. Each card has the following information, in this order:

<u>Station number</u>

$$\left.\begin{matrix} X \\ Y \\ Z \end{matrix}\right\} \quad \text{station coordinates (megameters);}$$

$\Phi$ station latitude (radians);
$\lambda$ station longitude (radians);
$\sigma$ station weight, or assumed accuracy (seconds of arc).

The coordinates must have decimal points punched.

```
9002  5.056291  2.716562  -2.775723  -.4530770  .4930620  4.0
9001  -1.535732  -5.167226  3.401154  .5658951  4.4234969  4.0
BINSTA
```

A code word is included on the station tape so that if the tape is not on the machine when the program needs it, a message to the operator is printed on-line.

A second numeric field <u>may</u> be punched on the BINSTA card. If this field is not zero, then the coordinates are not written on tape but stored in the machine. The following BINSTA card overrules this operation. If this field is zero and no coordinates follow (that is, if the next card is blank), then the coordinates are taken from the binary tape. This allows the user to use the binary tape on one orbit, read in the coordinates for the next orbit, then use the tape for a third orbit.

The binary tape consists of one long record terminated by an EOF. The first word on the tape is the code word 22314562632l in octal, that is, the binary equivalent of BINSTA. The rest of the record is made up of 8-word groups:

| Word | Contents | | Units |
|---|---|---|---|
| 1 | Weight as a floating-point number; the address contains the station number | | Radians |
| 2 | X | station coordinates | Megameters |
| 3 | Y | | |
| 4 | Z | | |
| 5 | sin ($\Phi$) | $\Phi$ = station astrometric latitude | |
| 6 | cos ($\Phi$) | | |
| 7 | sin ($\lambda$) | $\lambda$ = station longitude | |
| 8 | cos ($\lambda$) | | |

The stations are sorted in ascending order, according to station number.

5. The I∅TAPE card

This card enables the user to change the tape units at run time. It has two data fields, the first designating the logical function to be changed and the second indicating the unit to which it is to be changed. The unit designation may be either symbolic (e.g., A3) or numeric (e.g., BELL SYS 3-4).

```
I∅TAPE SYS∅UT  B3
I∅TAPE   SYSIN  A4
```

The standard setting of the units is

| Code | Tape function | Nominal setting |
|---|---|---|
| SYSIN | Input tape | A2 or 2 |
| SYS∅UT | Output tape | A3 or 3 |
| SYSPL1 | Pool tape A | A4 or 4 |
| SYSPL2 | Pool tape B | B2 or 12 |
| SYSDPL | Duplicate tape | B3 or 13; see DUPLT, p. 44 |
| SYSPUN | Punch output tape | B4 or 14 |
| SYSBST | Binary station tape | B5 or 15 |

6. The SSWT card

This card allows the user to set the sense switches as internal (up or down) or external. For each sense switch there must be two integers, of which the first specifies the switch (1-6) and the second, its mode.

| Mode | State of switch |
|---|---|
| 0 | Internal up |
| 1 | Internal down |
| 2 | Interrogate the console for the status of the sense switch. |

The sense-switch settings operate as follows:

| Switch | Normal mode | Remark |
|---|---|---|
| 1 | External up | If down, information--such as current satellite name, iteration count, and diagnostics--is printed on-line. |
| 2 | Ignored | |
| 3 | External up | If down, the current file is stopped; the orbit is written out; and the next control card is read. If this switch is not returned to the up position before the next satellite is processed, it will also be terminated. |
| 4 | Ignored | |
| 5 | External up | If down, the normal input is from the card-reader. |
| 6 | Internal up | If down, each record written on A3 is also printed on-line. This is not recommended, as it slows the program down. |

```
SSWT 1 0 2 1 3 2
```

7. The NPRINT card

With this card the program never references the printer but logically disconnects it and overrules printing diagnostics.

```
NPRINT
```

8. The PRINT card

The PRINT card logically reconnects the printer, say after an NPRINT card has been used.

```
PRINT
```

9. The GØ, GØBACK, and DUPLT cards

These three control cards are related. Several DØI runs often use the same set of observations. This set of cards allows the user to input the observations once and use them several times.

When the DUPLT card is placed before the observation cards on the input tape, it causes all observations of the next satellite to be written on a pool tape (SYSDPL, which is usually B3). This file is processed in the usual manner.

If the user wants some other tape unit, he punches that tape number in the second field on the card.

The two cards

```
DUPLT
IØTAPE SYSDPL B2
```

have the same effect as:

```
DUPLT B2
```

When the observations are to be reused, say after another set of elements has been input, the GØ card will cause GØBACK to be written on SYSDPL, this tape to be rewound, and the input to be taken from it. In the deck set-up for this second file, the GØ card replaces the observation cards. GØ cards may be used several times.

```
GØ
```

The GØBACK card causes the program to read the next record from SYSIN. The user generally need not concern himself with the GØBACK card, since it is automatically written on SYSDPL. This data tape is, however, written in BCD in order that it may be prepared off-line. In off-line preparation, the GØBACK card should be the last one written on the tape.

```
GØBACK
```

10. The AGAIN card

This card allows the program to redefine the coefficients to be varied and to proceed with a DØI run on the observations already read in. Since these observations must be on the SYSDPL tape, a DUPLT card must precede them (see the DUPLT control card). As with the element-definition card, any alphabetic information punched on this card will be printed on the output listing. The elements are redefined by using the same P3110 S100, etc., but they are not followed with the numerical values. The form of the equations and the number of coefficients defined must agree exactly; otherwise the program rejects the card and proceeds to the end-of-run procedures. N.B.: This card assumes that a DUPLT card has been previously sent; it has the effect of a GØ card. The definition must be followed by a trailing numeric field.

```
P100 P100  P00  P00  P31100
AGAIN
```

11. The BSF card

This card causes the tape defined by the second field to backspace one file. The tape may be designated by its logical function, its symbolic name, or its number (e.g., BESYS 3). In the following example the three cards are equivalent:

```
BSF 13
BSF B3
BSF SYSDPL
```

12. The BSR card

This card instructs the tape defined by the second field to backspace the number of records indicated in the third field.

13. The RUN card

Use of this card causes the tape designated by the second field to be rewound and unloaded. These three cards are equivalent:

```
RUN 2
RUN A2
RUN SYSIN
```

14. The REW card

This card causes the tape designated by the second field to be rewound. These three cards are equivalent:

```
REW 4
REW A4
REW SYSPL1
```

15. The WEF card

The WEF card causes an EOF (end of file) to be written on the tape designated by the second field. These three cards are equivalent:

```
WEF 3
WEF A3
WEF SYSØUT
```

16. The PAUSE card

This card, entirely printed on the on-line printer, causes the program to halt.

```
PAUSE        THIS MAY BE A MEANINGFUL MESSAGE
```

17. The SYSTEM card

This should be the last card of any run, as it initiates final clean-up of buffers, push-down lists, etc. It then terminates the job with an EOF on the SYSØUT tape and returns control to the monitor system. The current version of the program rewinds A1 (the system tape) and loads the first record. Returns to more exotic systems can easily be made by means of a small patch.

```
SYSTEM
```

18. The STAVAR card

The DØI program can treat the station coordinates as well as the orbital elements as unknowns and can compute corrections to them. All three coordinates must be improved. The STAVAR card defines, by number, the stations to be improved. Groups of station numbers are separated by zeros. The program can solve for station coordinates only, for orbital elements only, or for any combination of these. It can also compute one set of corrections (dx, dy, dz) for several stations (e.g., the stations of the same geodetic datum), using the observations from all those stations. Free-field conventions, of course, also apply. If more than one card is needed to define the stations, successive cards must have STAVAR punched as the first field. Since a new set of elements redefines the unknowns, after a run with stations as unknowns, the user must use a STAVAR card, with no stations listed on it, if he does not wish to vary the stations.

```
STAVAR  9001 9002 0 9003 0
```

```
STAVAR  9004  9005
```

The above example indicates that three sets of corrections are to be computed, one for 9001 and 9002, one for 9003, and one for 9004 and 9005. The stations, with their corrections, are printed on the output listing.

19. The CØNST card

The DØI utilizes certain basic physical constants, one or more of which the user may want to change. The CØNST card causes the program to use the number indicated in field 3 in place of the constant specified in field 2. The constants that can be changed with these cards are

| Symbol | Field 2 | Use | Initial value | Units |
|---|---|---|---|---|
| | MUØ | Constants for rotation, see RØTI and RØTII | 6.1991868E-7 | |
| | MUT | | 1.854875E-12 | |
| | ATZ | Constants for converting Al time to UT | -12.154E-6 | |
| | ATØ | | -0.153791E-6 | |
| | ATT | | 0. | |
| J | J | 2nd harmonic term in Earth's potential | .0660546 | Produces a in megameters |
| GM | GM | Obvious | 274.5391 | Produces n in revolutions |

An example would be:

```
CØNST  GM  274.59270
```

These constants will remain in effect until a later CØNST card changes them. This card is printed on the output listing.

20. The RØTI, RØTII control cards (see page 2 and 5)

There are two options for reducing observations in $\alpha$, $\delta$ to direction cosines. The RØTII, which adds the optional rotation to all observation reductions, remains in effect until a RØTI is input. The RØTI card sets the rotations back to the normal mode A memorandum by Kozai (1961) explains more fully the meanings of these terms.

21. The MAR card

This card changes the DØI from a least-squares program to a precise mean anomaly residual program. The DØI regularly assigns a convergence limit and a maximum iteration count; if the user wishes to change them, he must so indicate in fields 2 and 3. The program sets the convergence at 1.E-6 revolutions in M and the maximum iterations at 60; these figures are used if there are no numbers on the card. The iteration count is printed with each observation. This card remains in effect for the next set of observations. Successive orbits must have their own MAR cards. If the convergence limit and iteration count do not appear, the values set previously are used.

The program proceeds, observation by observation, to compute dM (see page 54), then adds this into $M_0$ of the mean anomaly equation. It then recomputes dM and repeats this cycle until the computed $dM_i$ is less than the convergence limit or has been computed more times than the count limit.

The orbit definition of the satellite must <u>not</u> have any coefficients treated as unknowns. This MAR card appears on the output listing.

```
MAR  1.E-6  20
```

22. The MAXITR card

The least-squares iteration procedure sets a maximum of 20 iterations. If this must for some reason be changed, the MAXITR card sets a new limit, which is punched in the second field. At the end of the next run the iteration count is reset to 20.

23. The REITER card

This card, which should follow the last observation of a run, initiates another mode of operation. When the normal DØI run is completed, two more iterations are performed; these use residuals in du and dw and the final orbit from the normal run, rather than residuals in cos $\delta$ $d\alpha$, $d\delta$. The residual in the plane of the orbit is du; dw is perpendicular to the orbit (see Izsak, 1961b). This operation solves for the same unknowns here as the previous run. This method can be used only with observations of two quantities; it cannot be used if station coordinates were the unknowns on the previous run.

```
REITER
```

24. The PURETR card

This card has the same effect as the REITER card but causes the reduced observations to be written on SYSPUN in binary so that they may be used as input in another program. Replacing the REITER card, the PURETR is reproduced on the output listing.

The format of the binary card is

Word

1, address (observation number), decrement (satellite code).
N.B.: The decrement is split because the fourth octal digit has the column binary 7-9 punch needed to identify binary cards.
2, standard error of the observation (floating point in radians).

Words 3-4 contain the time, with the speed of light subtracted:

3, time in MJD
4, time in MJD (double-precision floating point).

Words 5-12 contain the values of the mean elements at the time:

5, argument of perigee, $\omega$ (radians);
6, argument of ascending node, $\Omega$ (radians);
7, inclination, i (radians);
8, eccentricity, e (dimensionless);
9, Mean anomaly, M (radians);
10, Eccentric anomaly, E, e.g., solution of Kepler's equation (radians);
11, semimajor axis, a, with second harmonic term (megameters);
12, mean motion, n, with second harmonic term (revolutions, days -1);
13, 14, 15 direction cosines, X,Y,Z of the satellite position, with the short-period perturbations caused by the second harmonic;
16, slant range, $\rho$, from observing station (megameters);
17, the observed right ascension, sin $\alpha$, with the appropriate reductions (the low-order bit contains the sign of cos $\alpha$);
18, the same information as above for declination, sin $\delta$;
19, residual in the plane of the orbit, du (seconds of arc);
20, residual perpendicular to the plane of the orbit, dw (seconds of arc);
21, residual in mean anomaly, dM (revolutions);
22, sine of the position angle, $\psi$ (the low-order bit contains the sign of the cos $\chi$);
23, sine of the sidereal angle, $\theta$ (the low-order bit contains the sign the cos $\theta$);
24, checksum.

Columns 73-80 contain in BCD the satellite code and the last four digits of the observation number. These columns may be interpreted to identify particular observations.

25. The PLØT card

A simple plotting routine incorporated in the program treats one 1403 printed page as a 100-50 grid (X axis = 100; Y axis = 50). The DØI can plot any calculated parameter as a function of any other calculated parameter. At present the program contains a list that includes time from the epoch of the elements (T), residuals in declination (DEC), residuals in right ascension (RA), and residuals in dM (DM). The list, however, is easily expanded.

The PLØT card contains 7 or 8 data fields:

1, BCD name PLØT;
2, BCD name of X-axis variable (e.g., T);
3, BCD name of Y-axis variable (e.g., DM);
4, Numerical value of left-hand side of X axis (floating point);
5, Numerical value of right-hand side of X axis (floating point);
6, Numerical value of bottom of Y axis (floating point);
7, Numerical value of top of Y axis (floating point);
8, Optional; take every n-th point (integer).

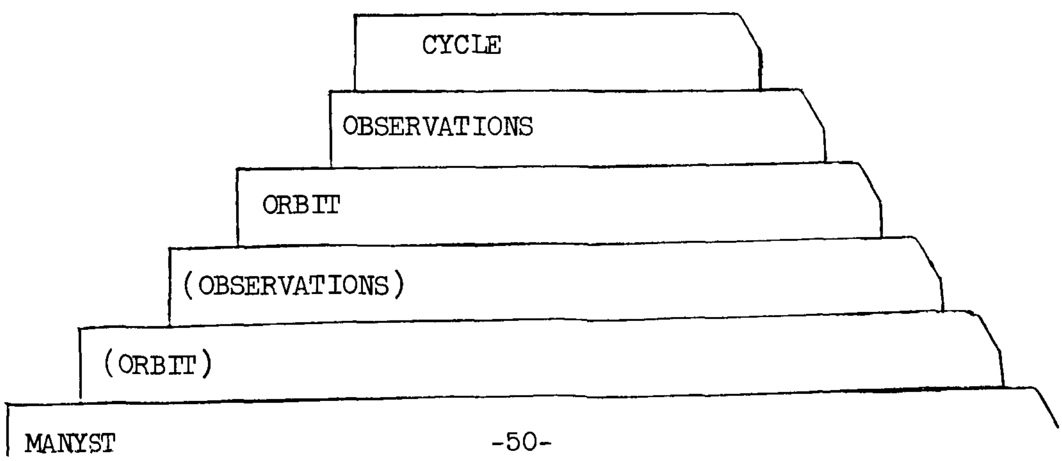
A limit of 400 points can be plotted. Some fields of observations, however, may contain more than this number; this card allows for a selection.

```
PLØT  T  DM  0.  5.  -1.E-4  1.E-4  3
```

This card, which may be used with any type run, will work on the stepping mode (see Epoch cards, page 33).

26. The MANYST card

Normally, for improving station coordinates only one satellite orbit may be used. For purposes of station determination only, however, many orbits (of the same or of different satellites) may be combined, with the restriction that no elements of any of the orbits may be treated as unknowns. (The stations to be improved must still be defined with STAVAR cards.) The MANYST card precedes the orbits and observations and causes the program to continue reading cards until a CYCLE card, which signals the end of input for the current run, is read. The observations must directly follow the corresponding orbit, even though other control cards may also be present.



27. The CYCLE card

This card, which must appear only in connection with a MANYST card, terminates the input for a run.

```
CYCLE
```

28. The LUNISØ card

This card instructs the program to add luni-solar perturbations to the orbital element. Since it affects only the following file, it must be input again before, for example, a REITER card, if the luni-solar perturbations are to be added for both of the iterations.

```
LUNISØ
```

A comment indicating that luni-solar perturbations have been added to the current orbit is printed.

29. The PLS card

This card instructs the program to print the first n computations of the luni-solar perturbations and the mean elements for the next satellite. Although it is primarily a debugging tool, it may be put to more general use.

The second field contains n; if this field is vacuous, then all values are printed. The luni-solar perturbations are computed only once for each observation.

```
PLS  20
```

30. The INVERS card

This card instructs the program to print the matrix inverse of the least-squares solution at the end of the next run. This matrix, which is unscaled, is sometimes referred to as the variance-covariance matrix.

```
INVERS
```

31. The CØRREL card

When this card is used, the program prints the correlation matrix of the least-squares solution at the end of the next run.

```
CØRREL
```

32. The MATRIX card

The MATRIX card, unscaled, instructs the program to print the least-squares matrix each time it is about to be inverted.

```
MATRIX
```

33. The PNDM card

This card instructs the program to produce on SYSPUN a BCD punch-card output suitable for input to the EAI dataplotter. It punches dM vs. T. Generally used with the MAR option, it may be used with any other option except the PURETR, which would produce a mixed-mode SYSPUN tape. The card format is described in Norris and Kawachi (1961). The station number is in the annotation field; the weight, in the comments field. The control card consists of four numeric fields:

1 Integer part of time origin (days, with decimal point)
2 Fractional part of time origin (days, with decimal point)
3 Scale factor for time scale
4 Scale factor for dM scale.

If any fields are missing, they are assumed to be 0, 0, 100., and 1.E5, respectively.

```
PNDM 37000. .0 10. 1.E4
```

34. The TIME card

When the program reads this card, it reads the on-line clock (accurate to .01 hour) and writes the time on the output.

```
TIME
```

35. The TIMER card

This card instructs the program to print the time whenever a new orbit is read in.

```
TIMER
```

36. The NØTIME card

The NØTIME mode, which is the normal mode, nullifies the TIMER card. TIME cards may be included even when the program is in the TIMER mode.

Blank cards should not appear in the deck except to terminate the station cards, as they bring the program to a halt. Pressing START will cause the program to reset and attempt to read the next control card. This is not, however, the recommended procedure for the final stop, nor is operator intervention. The SYSTEM and PAUSE cards have been provided for these purposes. The blank-card convention is retained for historical reasons and as an aid in debugging.

## Output

### Orbit

The input orbit is printed with each element designated by the degree of the polynomial followed by the n+1 coefficients, followed on succeeding lines by S's with three coefficients. Output consists of the same units, in the same order in which the material was input. The variable parameters are followed by the digit 1 and are output to the same accuracy as the input.

The final orbit is output in much the same manner, except that the improved quantities are output to the accuracy to which they were computed. The two digits following the new parameter represent the uncertainty in the last two digits of the number (e.g., 272.35125 27, which means that the number has an uncertainty of ± .00027). This uncertainty is computed from the variance-covariance matrix. If we are interested in the i-th variable, the uncertainty would be

$$\sigma \times \sqrt{a_{ii}} \text{ (converted to the relevant units),}$$

where $a_{ii}$ is an element of the variance-covariance matrix, and $\sigma$ is the over-all standard error of the solution.

If $\omega$ and M are variables, one can look at the uncertainties of the origin as $\omega_0 + M_0$. It is more meaningful, however, to consider $d(\Omega_0 + M_0)$, computed from

$$d(\Omega_0 + M_0) = \sigma \sqrt{a_{ii} \times (2\pi)^2 + 2 \times (2\pi) \times a_{ij} + a_{jj}} \,, \tag{59}$$

where $a_{ii}$ is the variance-covariance element of $M_0$; $a_{jj}$ is the element of $\omega_0$; and $a_{ij}$ is the off-diagonal element common to both. This is to say that $M_0$ is the i-th variable, and $\omega_0$ the j-th.

If station coordinates, stated in units of megameters, are variables, the original X,Y,Z coordinates are written out. When the solution is known, it is written out with the corrections and the uncertainties, both represented in meters. The values of the elements at the epoch, including contributions of any sine terms, are written out in the following units:

| Name | Units | Symbol |
|---|---|---|
| Perigee | degrees | $\omega$ |
| Argument of node | degrees | $\Omega$ |
| Inclination | degrees | i |
| Eccentricity | dimensionless | e |
| Anomaly | revolutions | M |
| Mean motion | revolutions/day | n |
| Semimajor axis | megameters | a |
| Perigee distance | megameters | q |
| Perigee distance from Earth's surface | megameters | q' |

where (Jacchia, private communication)

$$\left.\begin{aligned} q' &= q - R_e + .02144 \sin^2 \Phi'; \\ \sin \varphi' &= \sin i \sin \omega; \text{ and} \\ R_e &= 6.378388 \text{ megameters.} \end{aligned}\right\} . \tag{60}$$

Each observation is printed separately; some are omitted, either because of a USE ØMIT control card or because station coordinates were not included on the binary station tape.

From left to right across the page, one will find:

(1) Leading minus sign, indicating that this observation was rejected from the least-squares solution on the last iteration. In a MAR run a minus sign denotes that the maximum number of iterations allowed failed to produce convergence.

(2) OBS field, which contains the 5-digit observation number but does not include the satellite designation; STA field, which represents the station number; RS (s) field, which is the assumed accuracy of the observation used to weight the observation (in seconds of arc).

(3) TIME (UT), indicating the time of the observation, with A1 to UT1 conversion added if appropriate and travel time (the speed of light) subtracted.

(4) RHO field, the topocentric distance from the station to the satellite (in megameters).

(5) MA field, representing the mean anomaly (in revolutions) of the observation. This field is especially important for discovering errors in the run. A good least-squares solution is critically dependent on good distribution in the mean anomaly.

(6) The DM (REV) field, representing the error in mean anomaly expressed as a derivative (in revolutions). The value of dM is computed by

$$dM = \frac{\delta r}{\left|\frac{\partial \bar{r}}{\partial M}\right|} , \tag{61}$$

but $\delta r$ is defined differently for range, direction, and position observations.

If we define the unit vector along the orbit as

$$\tau = \frac{\partial \bar{x}}{\partial M} \Bigg/ \frac{\partial x}{\partial M} ,$$

then for range observations (see figure 7)

$$\delta r = \frac{r \left| \overline{\Delta r} \right|}{\bar{r} \times \hat{\tau}} .$$

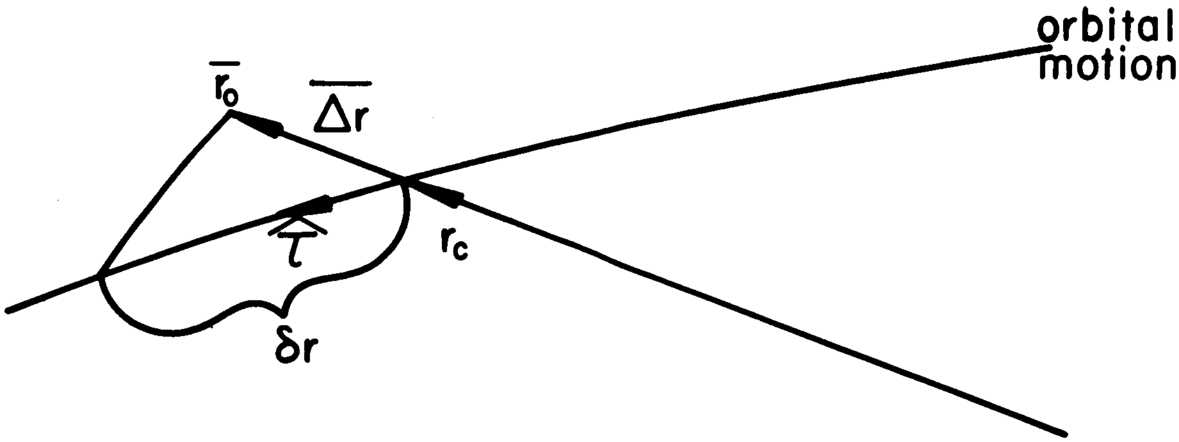



Figure 7.--Value of $\delta r$ for range observations.

For direction observations (see figure 8)

$$\delta r = \frac{\Delta r \cdot \hat{\tau}}{1 - \left(\frac{\vec{r}_c \cdot \hat{\tau}}{r}\right)^2} \quad .$$

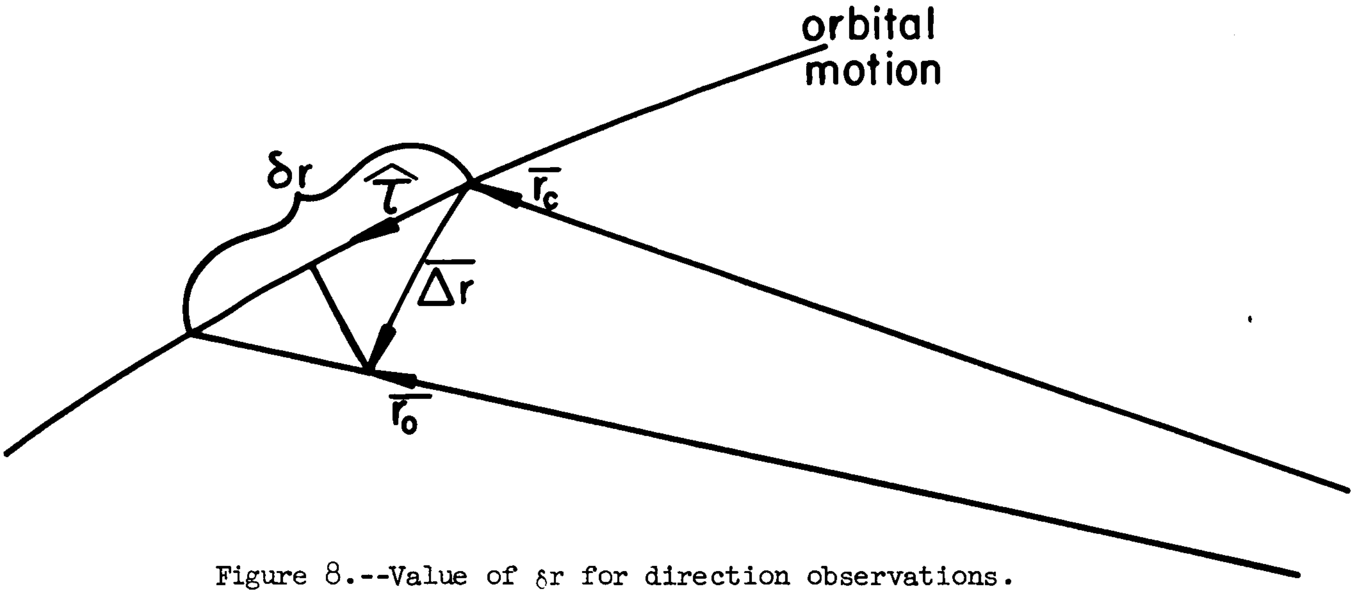



Figure 8.--Value of $\delta r$ for direction observations.

For position observations (see figure 9)

$$\delta r = \overline{\Delta r} \cdot \hat{\tau} \quad .$$

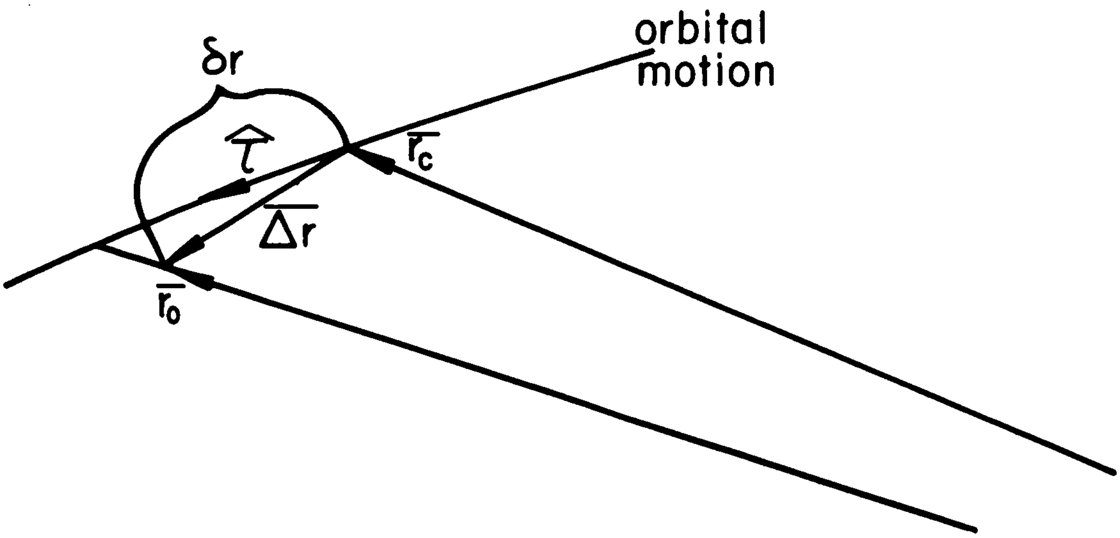



Figure 9.--Value of $\delta r$ for position observations.

Then, for all cases, if

$$dM = \frac{\partial M}{\partial r} \delta r ,$$

and we have $\overline{\partial r}/\partial M$ (see equation (29), page 18),

$$dM = \frac{\delta r}{\left|\frac{\partial \bar{r}}{\partial M}\right|} .$$

(7) DEC (S), RA (S), which represent the residual of the observation, in seconds of arc. All residuals are expressed in $d\delta$, $\cos \delta d\alpha$.

(8) RANGE (M), the residual in range, printed in meters if range is observed.

The basic statistics of each iteration are printed. The notation ITER. 0 132.13 104 for example, means that in the 0-th iteration the standard error was 132.13 and 104 observations were used. It is important to realize that this last number is the number of observations, not the number of observed quantities.

Diagnostics and Error Messages

These messages are printed on-line under SSW1 control and on SYSØUT.

(1) ILLEGAL TAPE SELECTED...UNIT XX XR4 = XXXXX

This means that one of the system-tape functions has been changed to a nonexistent unit, represented by a number greater than 20, or to the system tape A1. The two-digit tape number and the contents of index register 4 aid the user in discovering his folly. All I-Ø functions are checked in this way, and here the job usually terminates.

(2) What looks very much like a console dump is actually an unclassified error. Miscellaneous STR instructions, as well as error returns from subroutines, can cause this. The program generally tries to continue, but the user must consult the listing in order to interpret the cause of this error. Most likely it is a bug in the program, such as a wild stop or a loop, which causes the operator to enter STR in the keys to get the program off the machine (SØP at SAO). Index register 4 tells where the program transferred to the error routine, or in the case of a trap, to location 2, which itself transfers to the error routine (the program listing is well annotated). The address of location 0, containing the position of the trap, is in the address part of the indicators, and index register 4 at the time of the trap appears in the decrement part of the indicators.

(3) THE FOLLOWING STATION COORDINATE CARD IS INCOMPLETE AND WON'T BE USED

This means that there were not enough data fields on the station coordinate card.

(4) TOO MANY SATELLITES, OMIT PREVIOUS SAT, PROCEED

In the many-satellite mode (see MANYST control card) there is an upper limit to the number of different orbits allowed. Set within the program at 30, this can be changed by reassembling. The program tries to continue, processing the observations already read in, as if a CYCLE card had been read. This means that the following CYCLE card will be treated as an error.

(5) THE FOLLOWING RECORD IS INCORRECT AND HAS NOT BEEN USED

Simple checking should be performed on input cards and any inconsistencies should be marked as errors. A card so flagged usually has no effect on the program. Among the more common errors are

(a) incorrect number of data fields;

(b) satellite number not corresponding with observation number (this is the most common);

(c) modal errors, such as a CYCLE card not preceded by a MANYST card or a USEWMX card used in a REITER run;

(d) arithmetic errors in the editing and reduction of the observation cards (this generally means an error on the card, such as an ALT, AZ observation with the RA, DEC code);

(e) more than one MANYST card, or one used with an incorrect option (many options, including GØ, GØBACK, DUPLT, MANYST, USEWMX, cannot be used with MANYST cards).

(6) AGAIN CARD FAULTY, DELETED, CONTINUED

If the element definition on the AGAIN card does not exactly correspond in number with the elements previously defined, it is rejected.

(7) REFERENCE TO PUSHDOWN LIST 1 CANNOT BE MADE

This is either a machine error or a bug in the program.

(8) ITER. 1 1238 81

This is not a diagnostic but a detected error. Printed on-line under SSW1 control, it is meant to provide on-line monitoring. For runs with few observations, the print-cycle time can be large in comparison with the computing time, as each iteration is printed. The satellite name is also printed when read if SSW1 is down.

(9) NO OBSERVATIONS--INITIAL RESIDUALS

All the observations were rejected, and the program cannot continue with this solution. The residuals corresponding to the original elements are printed. The program ignores observations whose station coordinates are not included.

(10) ECC. = 1+ OR NEG.--INITIAL RESIDUALS

If at any time the correction to the eccentricity causes its value in the range to become < 0 or > 1, the file is terminated. The residuals corresponding to the original elements are then printed. Poor eccentricity fit generally means either wrong form of function for the eccentricity or, more likely, poor distribution of observations.

(11) NOT CONVERGING--INITIAL RESIDUALS

The program allows only 20 iterations. The current residuals are printed.

(12) SINGULAR MATRIX--INITIAL RESIDUALS

If the least-squares matrix cannot be inverted, this diagnostic occurs, and the residuals corresponding to the original elements are printed. This can happen if the distribution of observations is poor and a singular set occurs. Although this is not unlikely with a large number of unknowns, it usually happens when station coordinates are being improved and one of the stations fails to report any observations.

(13) DIVERGING--INITIAL RESIDUALS

If the standard error increases on four consecutive iterations, it is said to be diverging. The residuals corresponding to the original elements are printed.

(14) ERROR IN ORIGINAL ELEMENTS, NO INITIAL RESIDUALS

If the initial elements cannot be used, say because the eccentricity was negative, this diagnostic is printed.

(15) MACHINE ERROR, PRESS START TO PROCEED WITH NEXT CASE

This message occurs occasionally. It may be a bug in the program, but the program usually runs without further difficulty.

(16) PLEASE MOUNT DØI BINARY STATION TAPE PLEASE HIT START THANKS

If the binary station tape cannot be recognized, this message is printed on-line.

DØI-3 was written for a 32K 7090 with two data channels and 10 tape drives. No special 7090 features are employed, and we hope that the IØ is independent of the type of tape drive and the internal machine speed. Since the program is independent of the system IØ, it need not be run under the latter. It is an "absolute" program, in the sense that the binary deck has an absolute location assigned.

The programming has been done exclusively in BEFAP (MACRØ FAP); the program was written and debugged under BESYS3. We made extensive use of the Macro language and other features peculiar to this assembler, including the modification of the assembler itself.

Although the program methods are somewhat restrictive to change, there are areas that may easily be expanded (for instance, the control dictionary and the pushdown list of operations). Hopefully, the centralized IØ will make change to IØ monitors a simple matter.*

Following is a list of some of the more important library subroutines assembled into the program. It is not a complete list, as the heavily subroutined program has several routines peculiar to the DØI-3 itself.

| Subroutine | Function |
|---|---|
| GIØ | Input-output with inversion and IØ buffer supervisor |
| SQRT | Extracts square root |
| LN-S820LN-LAS820 | Obtains natural log |
| EXP-S816EX-LAS816 | Obtains exponentiation |
| ARSN-ARCØ CLASC1 | Extracts arc sin or arc cos |
| INV-UAINV1 | Matrix inversion |
| SIN-CØS-IBSIN1 | Obtains sin or cos |
| FSIN-FCØS | Fast sin or cos routine (accurate to 5 figures) |
| CRT1-MUCRT1 | Extracts cube root |

*The IØ philosophy and interpretation are completely detailed in Gaposchkin (1963). Since the data flow in this program is built around this GIØ routine, we recommend that any interested person consult this reference.

## Symbols for DØI Write-up

subscript O observed
C computed

subscript

$\vec{R}$ = station coordinates

$\vec{R} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ .

$\hat{\rho}_h$ horizon system

$\hat{\rho}_g$ geodetic geocentric

$\hat{\rho}_s$ inertial geocentric or sidereal system

$\vec{r}$ = satellite position (geocentric)

$\hat{r}$ direction cosine or unit vector

$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ r = magnitude of $\vec{r}$ $\therefore$ $\vec{r} = r\,\hat{r}$

$\vec{\rho}$ = topocentric satellite position

$\hat{\rho}$ direction cosine or unit vector

$\hat{\rho} = \begin{pmatrix} \ell \\ m \\ n \end{pmatrix} = \bar{\ell}$ (sometimes)

t = time in days T = time in years

$\tau$ = epoch

$\omega$ = perigee

$\Omega$ = node

i = inclination

e = eccentricity

M = mean anomaly

n = mean motion = $\dot{M}$

no subscript is satellite
M subscript is moon ($\omega_M$)
$\odot$ subscript is sun ($\omega_\odot$)
E subscript is earth ($\omega_E$)

a = semimajor axis

$\theta$ = sidereal time

$\Phi$ = astrometric latitude of station (altitude of the north celestial pole)

$\lambda$ = longitude of the station

E = eccentric anomaly

v = true anomaly

$\ell = \omega + V$

Note that $\rightharpoonup$ means vector

$\wedge$ means unit vector

$r$ scalar

$\vec{r}$ vector

$\hat{r}$ unit vector, such that $\vec{r} = r\,\hat{r}$

$$r = |\vec{r}|$$

$$\vec{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \qquad \hat{r} = \frac{1}{r}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{\sqrt{x^2 + y^2 + z^2}} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$r = \sqrt{x^2 + y^2 + z^2}$$

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# APPENDIX 1

## Expressions of Lunar Perturbations

$\dot{I}_M \approx 0 \quad \dot{\Omega}_M \approx 0$ $\qquad n_M = 13\overset{\circ}{.}23 \text{ day}^{-1}$

$\dot{\lambda}_M = n_M \quad \dot{\hat{\Omega}} = -\Omega_1 \quad \dot{\omega} = \omega_1 \qquad \underline{\Omega' = \Omega_1/n_M} \qquad \underline{\omega' = \omega_1/n_M}$

$\delta a = 0$

$$
\begin{aligned}
\delta e = \frac{15}{8}\frac{\nu_M^2}{n_M n} e\sqrt{1-e^2} \Big\{ & -6\gamma^2\sigma^2 \cdot \gamma_M^2\sigma_M^2 \cos(2\lambda_M + 2\omega)/(1+\omega') \\
& + 4\gamma\sigma^3 \cdot \gamma_M^3\sigma_M \cos(2\lambda_M + \hat{\Omega} + 2\omega)/(1 - \Omega'/2 + \omega') \\
& + 4\gamma^3\sigma \cdot \gamma_M\sigma_M^3 \cos(2\lambda_M - \hat{\Omega} + 2\omega)/(1 + \Omega'/2 + \omega') \\
& - 6^4 \cdot \gamma_M^4 \cos(2\lambda_M + 2\hat{\Omega} + 2\omega)/(1 - \Omega' + \omega') \\
& - \gamma^4 \cdot \sigma_M^4 \cos(2\lambda_M - 2\hat{\Omega} + 2\omega)/(1 + \Omega' + \omega') \\
& + 6\gamma^2\sigma^2 \cdot \gamma_M^2\sigma_M^2 \cos(2\lambda_M - 2\omega)/(1 - \omega') \\
& + 4\gamma^3\sigma \cdot \gamma_M^3\sigma_M \cos(2\lambda_M + \hat{\Omega} - 2\omega)/(1 - \Omega'/2 - \omega') \\
& + 4\gamma\sigma^3 \cdot \gamma_M\sigma_M^3 \cos(2\lambda_M - \hat{\Omega} - 2\omega)/(1 + \Omega'/2 - \omega') \\
& + \gamma^4 \cdot \gamma_M^4 \cos(2\lambda_M + 2\hat{\Omega} - 2\omega)/(1 - \Omega' - \omega') \\
& + \sigma^4 \cdot \sigma_M^4 \cos(2\lambda_M - 2\hat{\Omega} - 2\omega)/(1 + \Omega' - \omega') \Big\}
\end{aligned}
$$

$I = 5\overset{\circ}{.}145 \qquad \epsilon = 23\overset{\circ}{.}444$

$\Omega = 148\overset{\circ}{.}753 - 0\overset{\circ}{.}0529539d$

$\lambda_M = \eta_M + 53\overset{\circ}{.}663 + 13\overset{\circ}{.}2293504d$

$$\gamma_M \cos\frac{\Omega_M + \eta_M}{2} = \cos\frac{I+\epsilon}{2}\cos\frac{\Omega}{2}$$

$$\gamma_M \sin\frac{\Omega_M + \eta_M}{2} = \cos\frac{I-\epsilon}{2}\sin\frac{\Omega}{2}$$

$$\sigma_M \cos\frac{\Omega_M - \eta_M}{2} = \sin\frac{I+\epsilon}{2}\cos\frac{\Omega}{2}$$

$$\sigma_M \sin\frac{\Omega_M - \eta_M}{2} = \sin\frac{I-\epsilon}{2}\sin\frac{\Omega}{2}$$

$\hat{\Omega} = \Omega_M - \Omega$

Epoch: 2437500.5 = July 20.0, 1961

$\gamma = \cos i/2 \qquad \sigma = \sin i/2$

Perturbations in radians:

$$\frac{\nu_M^2}{n_M} = .2783 \times 10^{-2}/\text{day}$$

n in radians/day

$\Omega_1$, $\omega_1$ in degrees/day

$\Omega' = \Omega_1/13.23$
$\omega' = \omega_1/13.23$

$$
\begin{aligned}
\delta\omega = \frac{3}{16}\frac{\upsilon_M^2}{n_M n}\frac{1}{\sqrt{1-e^2}}\cdot \Big\{ & + 6[4(1 - 5\sigma^2 + 5\sigma^4) + e^2]\cdot \gamma_M^2\sigma_M^2 \sin 2\lambda_M \\
& + \gamma^{-1}\sigma^{-1}[2(1 - 22\sigma^2 + 60\sigma^4 - 40\sigma^6) + 3e^2(1 - 2\sigma^2)]\cdot \gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega})/(1 - \Omega'/2) \\
& - \gamma^{-1}\sigma^{-1}[2(1 - 22\sigma^2 + 60\sigma^4 - 40\sigma^6) + 3e^2(1 - 2\sigma^2)]\cdot \gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega})/(1 + \Omega'/2) \\
& - [2(1 - 10\sigma^2 + 10\sigma^4) + 3e^2]\cdot \gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega})/(1 - \Omega') \\
& - [2(1 - 10\sigma^2 + 10\sigma^4) + 3e^2]\cdot \sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega})/(1 + \Omega') \\
& + 15[4\gamma^2\sigma^2 - e^2]\cdot \gamma_M^2\sigma_M^2 \sin(2\lambda_M + 2\omega)/(1 + \omega') \\
& - 5\gamma^{-1}\sigma[8\gamma^2\sigma^2 - e^2(3 - 2\sigma^2)]\cdot \gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega} + 2\omega)/(1 - \Omega'/2 + \omega') \\
& - 5\gamma\sigma^{-1}[8\gamma^2\sigma^2 - e^2(1 + 2\sigma^2)]\cdot \gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega} + 2\omega)/(1 + \Omega'/2 + \omega') \\
& + 5\sigma^2[2\sigma^2 - e^2]\cdot \gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega} + 2\omega)/(1 - \Omega' + \omega') \\
& + 5\gamma^2[2\gamma^2 - e^2]\cdot \sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega} + 2\omega)/(1 + \Omega' + \omega') \\
& + 15[4\gamma^2\sigma^2 - e^2]\cdot \gamma_M^2\sigma_M^2 \sin(2\lambda_M - 2\omega)/(1 - \omega') \\
& + 5\gamma\sigma^{-1}[8\gamma^2\sigma^2 - e^2(1 + 2\sigma^2)]\cdot \gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega} - 2\omega)/(1 - \Omega'/2 - \omega') \\
& + 5\gamma^{-1}\sigma[8\gamma^2\sigma^2 - e^2(3 - 2\sigma^2)]\cdot \gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega} - 2\omega)/(1 + \Omega'/2 - \omega') \\
& + 5\gamma^2[2\gamma^2 - e^2]\cdot \gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega} - 2\omega)/(1 - \Omega' - \omega') \\
& + 5\sigma^2[2\sigma^2 - e^2]\cdot \sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega} - 2\omega)/(1 + \Omega' - \omega') \Big\}
\end{aligned}
$$

$$\delta\Omega = \frac{3}{16}\frac{\nu_M^2}{n_M n}\frac{2+3e^2}{\sqrt{1-e^2}}\left\{ \begin{aligned} & - 6(1-2\sigma^2)\cdot\gamma_M^2\sigma_M^2 \sin(2\lambda_M)/(1) \\ & - \gamma^{-1}\sigma^{-1}(1-8\sigma^2+8\sigma^4)\cdot\gamma_M^3\sigma_M \sin(2\lambda_M+\hat{\Omega})/(1-\Omega'/2) \\ & + \gamma^{-1}\sigma^{-1}(1-8\sigma^2+8\sigma^4)\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M-\hat{\Omega})/(1+\Omega'/2) \\ & + (1-2\sigma^2)\cdot\gamma_M^4 \sin(2\lambda_M+2\hat{\Omega})/(1-\Omega') \\ & + (1-2\sigma^2)\cdot\sigma_M^4 \sin(2\lambda_M-2\hat{\Omega})/(1+\Omega') \end{aligned}\right\}$$

$$+ \frac{15}{16}\frac{\nu_M^2}{n_M n}\frac{e^2}{\sqrt{1-e^2}}\left\{ \begin{aligned} & + 3(1-2\sigma^2)\cdot\gamma_M^2\sigma_M^2 \sin(2\lambda_M+2\omega)/(1+\omega') \\ & - \gamma^{-1}\sigma(3-4\sigma^2)\cdot\gamma_M^3\sigma_M \sin(2\lambda_M+\hat{\Omega}+2\omega)/(1-\Omega'/2+\omega') \\ & - \gamma\sigma^{-1}(1-4\sigma^2)\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M-\hat{\Omega}+2\omega)/(1+\Omega'/2+\omega') \\ & + \sigma^2\cdot\gamma_M^4 \sin(2\lambda_M+2\hat{\Omega}+2\omega)/(1-\Omega'+\omega') \\ & - \gamma^2\cdot\sigma_M^4 \sin(2\lambda_M-2\hat{\Omega}+2\omega)/(1+\Omega'+\omega') \\ & + 3(1-2\sigma^2)\cdot\gamma_M^2\sigma_M^2 \sin(2\lambda_M-2\omega)/(1-\omega') \\ & + \gamma\sigma^{-1}(1-4\sigma^2)\cdot\gamma_M^3\sigma_M \sin(2\lambda_M+\hat{\Omega}-2\omega)/(1-\Omega'/2-\omega') \\ & + \gamma^{-1}\sigma(3-4\sigma^2)\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M-\hat{\Omega}-2\omega)/(1+\Omega'/2-\omega') \\ & - \gamma^2\cdot\gamma_M^4 \sin(2\lambda_M+2\hat{\Omega}-2\omega)/(1-\Omega'-\omega') \\ & + \sigma^2\cdot\sigma_M^4 \sin(2\lambda_M-2\hat{\Omega}-2\omega)/(1+\Omega'-\omega') \end{aligned}\right\}$$

$$
\begin{aligned}
\delta i = -\frac{3}{8}\frac{\nu_M^2}{n_M n}\frac{2+3e^2}{\sqrt{1-e^2}} \Big\{ & + (1-2\sigma^2)\cdot\gamma_M^3\sigma_M \cos(2\lambda_M+\hat{\Omega})/(1-\Omega'/2) \\
& + (1-2\sigma^2)\cdot\gamma_M\sigma_M^3 \cos(2\lambda_M-\hat{\Omega})/(1+\Omega'/2) \\
& - \gamma\sigma\cdot\gamma_M^4 \cos(2\lambda_M+2\hat{\Omega})/(1-\Omega') \\
& + \gamma\sigma\cdot\sigma_M^4 \cos(2\lambda_M-2\hat{\Omega})/(1+\Omega') \Big\}
\end{aligned}
$$

$$
\begin{aligned}
-\frac{15}{8}\frac{\nu_M^2}{n_M n}\frac{e^2}{\sqrt{1-e^2}} \Big\{ & - 3\gamma\sigma(1-2\sigma^2)\cdot\gamma_M^2\sigma_M^2 \cos(2\lambda_M+2\omega)/(1+\omega') \\
& + \sigma^2(3-4\sigma^2)\cdot\gamma_M^3\sigma_M \cos(2\lambda_M+\hat{\Omega}+2\omega)/(1-\Omega'/2+\omega') \\
& + \gamma^2(1-4\sigma^2)\cdot\gamma_M\sigma_M^3 \cos(2\lambda_M-\hat{\Omega}+2\omega)/(1+\Omega'/2+\omega') \\
& - \gamma\sigma^3\cdot\gamma_M^4 \cos(2\lambda_M+2\hat{\Omega}+2\omega)/(1-\Omega'+\omega') \\
& + \gamma^3\sigma\cdot\sigma_M^4 \cos(2\lambda_M-2\hat{\Omega}+2\omega)/(1+\Omega'+\omega') \\
& + 3\gamma\sigma(1-2\sigma^2)\cdot\gamma_M^2\sigma_M^2 \cos(2\lambda_M-2\omega)/(1-\omega') \\
& + \gamma^2(1-4\sigma^2)\cdot\gamma_M^3\sigma_M \cos(2\lambda_M+\hat{\Omega}-2\omega)/(1-\Omega'/2-\omega') \\
& + \sigma^2(3-4\sigma^2)\cdot\gamma_M\sigma_M^3 \cos(2\lambda_M-\hat{\Omega}-2\omega)/(1+\Omega'/2-\omega') \\
& - \gamma^3\sigma\cdot\gamma_M^4 \cos(2\lambda_M+2\hat{\Omega}-2\omega)/(1-\Omega'-\omega') \\
& + \gamma\sigma^3\cdot\sigma_M^4 \cos(2\lambda_M-2\hat{\Omega}-2\omega)/(1+\Omega'-\omega') \Big\}
\end{aligned}
$$

$$\delta M = -\frac{3}{4}\frac{\nu_M^2}{n_M n}(7 + 3e^2)\Big\{ + (1 - 6\sigma^2 + 6\sigma^4)\cdot\gamma_M^2\sigma_M^2 \sin 2\lambda_M$$

$$- 2\gamma\sigma(1 - 2\sigma^2)\cdot\gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega})/(1 - \Omega'/2)$$

$$+ 2\gamma\sigma(1 - 2\sigma^2)\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega})/(1 + \Omega'/2)$$

$$+ \gamma^2\sigma^2\cdot\gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega})/(1 - \Omega')$$

$$+ \gamma^2\sigma^2\cdot\sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega})/(1 + \Omega')\Big\}$$

$$-\frac{15}{8}\frac{\nu_M^2}{n_M n}(1 + e^2)\Big\{ + 6\gamma^2\sigma^2\cdot\gamma_M^2\sigma_M^2 \sin(2\lambda_M + 2\omega)/(1 + \omega')$$

$$- 4\gamma\sigma^3\cdot\gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega} + 2\omega)/(1 - \Omega'/2 + \omega')$$

$$- 4\gamma^3\sigma\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega} + 2\omega)/(1 + \Omega'/2 + \omega')$$

$$+ \sigma^4\cdot\gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega} + 2\omega)/(1 - \Omega' + \omega')$$

$$+ \gamma^4\cdot\sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega} + 2\omega)/(1 + \Omega' + \omega')$$

$$+ 6\gamma^2\sigma^2\cdot\gamma_M^2\sigma_M^2 \sin(2\lambda_M - 2\omega)/(1 - \omega')$$

$$+ 4\gamma^3\sigma\cdot\gamma_M^3\sigma_M \sin(2\lambda_M + \hat{\Omega} - 2\omega)/(1 - \Omega'/2 - \omega')$$

$$+ 4\gamma\sigma^3\cdot\gamma_M\sigma_M^3 \sin(2\lambda_M - \hat{\Omega} - 2\omega)/(1 + \Omega'/2 - \omega')$$

$$+ \gamma^4\cdot\gamma_M^4 \sin(2\lambda_M + 2\hat{\Omega} - 2\omega)/(1 - \Omega' - \omega')$$

$$+ \sigma^4\cdot\sigma_M^4 \sin(2\lambda_M - 2\hat{\Omega} - 2\omega)/(1 + \Omega' - \omega')\Big\}$$

APPENDIX 2

Expressions for Short-Period Perturbations (2nd harmonic)*

$$\delta L = \frac{J}{p^2}\left[ \frac{i}{2}\left\{\left(-1 + \frac{7}{6}\sin^2 i\right)\sin(2\omega + 2v) + e\left[\left(-1 + \frac{5}{3}\sin^2 i\right)\sin(2\omega + v) + \frac{(-1 + \sin^2 i)\sin(2\omega + 3v)}{3}\right]\right\} - \left\{\frac{1}{3}\left(-1 + \frac{3}{2}\sin^2 i\right)\left[\left(1 - \sqrt{1 - e^2}\right)\sin v \cos v + e\left(\frac{e}{1 + \sqrt{1 - e^2}}\right)^2 \sin v\right] + (v - M + e \sin v)\left(-2 + \frac{5}{2}\sin^2 i\right)\right\}\right]$$

$$\delta r = \frac{1}{3}\frac{J}{p}\left[\left(-1 + \frac{3}{2}\sin^2 i\right)\left(1 - \frac{1 - e\cos E}{\sqrt{1 - e^2}} + \frac{e}{1 + \sqrt{1 - e^2}}\cos v\right) + \frac{1}{2}\cos(2\omega + 2v)\sin^2 i\right]$$

$$\delta\Omega = \frac{J}{p^2}\cos i\left[-(V - M + e \sin V) + \frac{1}{2}\left\{\sin(2\omega + 2v) + e\left[\sin(2\omega + v) + \frac{1}{3}\sin(2\omega + 3v)\right]\right\}\right]$$

$$\delta i = \frac{1}{2}\sin i\left(\frac{J}{p^2}\cos\right)\left[\cos(2\omega + 2v) + e\left\{\cos(2\omega + v) + \frac{1}{3}\cos(2\omega + 3v)\right\}\right]$$

For definition of p, see equation (21), page 13.

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*Kozai (1959).

APPENDIX 3

## S. A. O. Calendar

### YEAR DAYS SINCE JANUARY 0, NORMAL YEAR

| JAN | FEB | MAR | APR | MAY | JUNE | JULY | AUG | SEPT | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 31 | 59 | 90 | 120 | 151 | 181 | 212 | 243 | 273 | 304 | 334 |

### YEAR DAYS SINCE JANUARY 0, LEAP YEAR

| JAN | FEB | MAR | APR | MAY | JUNE | JULY | AUG | SEPT | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 31 | 60 | 91 | 121 | 152 | 182 | 213 | 244 | 274 | 305 | 335 |

### SMITHSONIAN DAY FOR DAY 0 OF MONTH

| YEAR | JAN | FEB | MAR | APR | MAY | JUNE | JULY | AUG | SEPT | OCT | NOV | DEC |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1957 | 35838 | 35869 | 35897 | 35928 | 35958 | 35989 | 36019 | 36050 | 36081 | 36111 | 36142 | 36172 |
| 1958 | 36203 | 36234 | 36262 | 36293 | 36323 | 36354 | 36384 | 36415 | 36446 | 36476 | 36507 | 36537 |
| 1959 | 36568 | 36599 | 36627 | 36658 | 36688 | 36719 | 36749 | 36780 | 36811 | 36841 | 36872 | 36902 |
| 1960 | 36933 | 36964 | 36992 | 37024 | 37054 | 37085 | 37115 | 37146 | 37177 | 37207 | 37238 | 37268 |
| 1961 | 37299 | 37330 | 37358 | 37389 | 37419 | 37450 | 37480 | 37511 | 37542 | 37572 | 37603 | 37633 |
| 1962 | 37664 | 37695 | 37723 | 37754 | 37784 | 37815 | 37845 | 37876 | 37907 | 37937 | 37968 | 37998 |
| 1963 | 38029 | 38060 | 38088 | 38119 | 38149 | 38180 | 38210 | 38241 | 38272 | 38302 | 38333 | 38363 |
| 1964 | 38394 | 38425 | 38454 | 38485 | 38515 | 38546 | 38576 | 38607 | 38638 | 38668 | 38699 | 38729 |
| 1965 | 38760 | 38791 | 38819 | 38850 | 38880 | 38911 | 38941 | 38972 | 39003 | 39033 | 39064 | 39094 |
| 1966 | 39125 | 39156 | 39184 | 39215 | 39245 | 39276 | 39306 | 39337 | 39368 | 39398 | 39429 | 39459 |
| 1967 | 39490 | 39521 | 39549 | 39580 | 39610 | 39641 | 39671 | 39702 | 39733 | 39763 | 39794 | 39824 |
| 1968 | 39855 | 39886 | 39915 | 39946 | 39976 | 40007 | 40037 | 40068 | 40099 | 40129 | 40160 | 40190 |
| 1969 | 40221 | 40252 | 40280 | 40311 | 40341 | 40372 | 40402 | 40433 | 40464 | 40494 | 40525 | 40555 |
| 1970 | 40586 | 40617 | 40645 | 40676 | 40706 | 40737 | 40767 | 40798 | 40829 | 40859 | 40890 | 40920 |

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory. First issued to ensure the immediate dissemination of data for satellite tracking, the Reports have continued to provide a rapid distribution of catalogues of satellite observations, orbital information, and preliminary results of data analyses prior to formal publication in the appropriate journals.

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